Selling Signals

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Abstract

This paper studies a signaling model in which a strategic player determines the cost structure of signaling. A principal chooses a price schedule for a product, and an agent with a hidden type chooses how much to purchase as a signal to the market. When the market observes the price schedule, the principal charges monopoly prices, and the agent purchases less than the first-best. In contrast, when the market does not observe the price schedule, the principal charges lower prices, and the agent purchases more than in the observed case; those of the highest types purchase more than the first-best. In terms of payoffs, the principal gains lower profits, whereas the agent obtains higher utility than in the observed case. The model can be applied to schools choosing tuition, retailers selling luxury goods and media companies selling advertising messages.

1 Introduction

In classic signaling models, the sender's preference depends only on his intrinsic type. This paper investigates situations in which the signaling cost also depends on the choice made by a third-party strategic player. For example, when a student obtains education to signal his ability, the university sets the tuition; when a consumer purchases a luxury good to signal his wealth, the retailer chooses the price; when a seller incurs advertising expenses to signal

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a product's quality, the media company determines the cost of advertising messages. A key observation is that since signaling cost is endogenous, how receivers interpret and respond to the sender's signal will depend on whether they observe the third party's choice.

In this paper, we derive the optimal price schedule for a principal selling a product to an agent who is endowed with a hidden type and chooses how much to purchase as a signal to the market. The equilibrium depends critically on whether the market observes the price schedule. When the market observes the price schedule, the principal internalizes the agent's signaling incentive when screening the agent, leading to a downward distortion in quantity. In contrast, when the market does not observe the price schedule, the agent is more sensitive to price changes, since the market will attribute a difference in quantity to agent preference heterogeneity. This provides the principal with an incentive to lower prices. In equilibrium, the agent chooses a higher quantity and obtains higher utility than in the observed case, whereas the principal gains lower profits than in the observed case.

This paper has meaningful implications for the price transparency of goods that create signaling values for consumers. In the case of job market signaling, our model suggests that education is more costly and students are worse off when the tuition scheme (more precisely, the net prices for school) is observed by employers than otherwise. This implies that policies that improve the transparency of the net prices at colleges and universities, e.g., U.S. Code § 1015a,[1] may *unintentionally* raise education expenses and harm students. This is because these policies allow schools to commit to high prices and not dilute the signaling value of a high-cost education by means of fee waivers or financial aid.

In addition, our model implies that a signaling good will yield higher profits if the seller can make the price publicly observed and commit to it. This echoes some real-world business practices. For example, luxury brands, such as Louis Vuitton, Tiffany and Hermes, enjoy a reputation of never or very rarely being on sale. This helps the sellers better commit to high prices, thereby reinforcing the signaling values of luxury goods. In the advertising industry, the high costs of each year's Super Bowl commercials are widely reported, thereby enhancing the signaling value of these costly commercials; in China, the TV station CCTV broadcasts the auctions for its popular TV show commercials to accentuate their signaling values.

For the purpose of exposition, we present our model in conformity with the seminal work of Spence (1973) with productive education. We extend that model by adding a pre-signaling stage in which a school (*principal*) sets its tuition scheme and a worker (*agent*) chooses his

[1]Since 2011, American colleges and universities have been required to provide reasonable estimates of the net prices, including tuition, miscellaneous fees and personal expenses, that students will pay for school. See "U.S. Code § 1015a - Transparency in college tuition for consumers" for details.

education level to signal his privately known ability (*type*) to competing employers (*market*). In Section 3, as a reference point, we briefly revisit Spence's model by fixing tuition at zero. This is the case when schools are competitive and set the price equal to the marginal cost. In the least-cost separating equilibrium, all types except the lowest one choose more education than the first-best, as they attempt to separate themselves from lower types.

In Section 4, we introduce the school and study the case in which employers observe the tuition scheme. In the school-optimal separating equilibrium (which is also the least-cost separating equilibrium), all types except the highest one choose less education than the first-best. This result is in contrast to that of Spence's model. The downward distortion is due to the school's screening. Having a cost advantage in education, a higher type can secure higher utility than a lower type by choosing the same education level. To incentivize truth-telling, the school must leave information rents to the worker, meaning that the marginal profit is less than the social surplus. Thus, the school under-supplies education.

While this mechanism is similar to screening models such as Mussa and Rosen (1978), our model also incorporates signaling, which can mitigate the downward distortion caused by screening. To illustrate, suppose that employers can observe the worker's ability, thereby eliminating signaling. When a higher type imitates a lower type, he not only incurs a lower total cost than the latter but also obtains a higher wage due to his higher ability. The second effect means that the worker can extract more information rents from the school; thus, the screening distortion is worse compared to when signaling is present.

In Section 5, we turn to the case in which employers do not observe the tuition scheme. In the school-optimal separating equilibrium (which is also the least-cost separating equilibrium), the school sets lower tuition rates and the worker chooses more education than when employers observe the tuition scheme. This difference is driven by a *signal jamming effect*. Because employers cannot observe the actual cost of education, they do not know whether a difference in education level is caused by a tuition change or worker cost heterogeneity. For example, suppose that the school lowers tuition so that the worker obtains more education than in the initial state. When employers observe the tuition scheme, they cut wages, since any education level now corresponds to a lower-ability worker. This dampens the worker's demand for additional education. In contrast, when employers do not observe the tuition scheme, they do not adjust wages despite that tuition changes. Consequently, the demand for education is more elastic, making the price cut relatively more profitable for the school. In equilibrium, employers correctly anticipate the school's incentive to cut tuition and offer lower wages, as education is inflated. This reduces the worker's willingness to pay, and thus, the school achieves lower profits when employers do not observe the tuition scheme.

Since the school is worse off when employers do not observe the tuition scheme, one may wonder why the school does not disclose tuition to employers. The reason is that the school cannot credibly announce the price absent intervention such as mandatory disclosure. Note that once employers believe the school's announcement, the latter would secretly cut prices to make a profitable deviation. Such an observation may explain the fact that while the listed tuition at American colleges and universities is rising, these schools offer students various and inclusive forms of financial aid.[2] The rationale is that employers cannot easily observe the details of such financial aid and thus do not know the actual cost of education. By raising the published tuition while simultaneously reducing the undisclosed net prices through stipends, schools persuade employers that their students are smarter than is actually the case, thereby allowing the schools to collect higher revenues from students.

Finally, in Section 6, we discuss the application and extension of the model and conclude our paper. All omitted proofs are provided in the Appendix.

1.1 Related Literature

This paper is most closely related to the literature on signaling. The paper contributes to the literature on signaling games by allowing a strategic player to affect signaling cost. In classic signaling models (e.g., Spence 1973, Riley 1985, Milgrom and Roberts 1986, Bagwell and Riordan 1991, Bagwell and Bernheim 1996), with an exogenous cost function, signaling activity gives rise to over-investment in costly actions. Spence (1974), Ireland (1994) and Andersson (1996) suggest taxing signaling activity to undo the signaling effect to restore the first-best. The associated tax scheme is thus the welfare-maximizing tax on signals. In our model, when the market observes the price schedule, we solve for the profit-maximizing tax on signals, which "over-taxes" signaling and causes a downward distortion in quantity.

The paper is also closely related to the literature on screening. Screening models, such as Mussa and Rosen (1978) and Maskin and Riley (1984), typically assume that buyers derive intrinsic utility from consuming the seller's product. Our model differs in the sense that the product has further a signaling value, and a buyer's utility depends on the information that

[2] According to the reports by The College Board (www.collegeboard.org): "from 2007-08 through 2010-11, the percentage of institutional grant aid that helped to meet students' financial need at private nonprofit four-year colleges and universities ranged from a low of 90% to a high of 93%" (*Trends in Student Aid 2011*, The College Board); "between 2008-09 and 2013-14, the $3,800 increase (in 2013 dollars) in average institutional grant aid for first-time full-time students at private bachelor's institutions covered 95% of the $4,000 increase in tuition and fees" (*Trends in Student Aid 2016*, The College Board).

the product conveys. As such, our model contains both screening and signaling and clearly states the interaction between the two forces. Calzolari and Pavan (2006) study information disclosure in a sequential screening model. They show that the upstream principal leaves more information rents to the agent if she discloses information about the agent's type to the downstream principal. Analogously, in our model, the principal leaves more rents to the agent than she would otherwise if the market can observe the agent's type, which is perfect information disclosure. The difference is that the market in our model is competitive; thus, unlike in their model, the disclosure of the agent's type creates no value for the market.

The model is closest to Rayo (2013). This paper also considers a principal who sells a signal to an agent with a hidden type, assuming that the principal's mechanism is observed by the market. Whereas we assume additive separability in the market's action (e.g., wage) and the agent's type (e.g., ability), Rayo's adopts a multiplicative structure, and thus, the principal's revenue depends on whether the allocation of signal is separating or pooling; this necessitates the use of novel screening techniques. The contribution of our paper is to study the case in which the market cannot observe the principal's mechanism, and comparing this to the observed case as well as a variety of other benchmarks. This enables us to assess how the transparency of pricing affects the degree of signaling and welfare.

The unobserved tuition case belongs to the class of signal jamming models proposed by Fudenberg and Tirole (1986). For example, in Holmström (1999), the labor market cannot distinguish the impact of the worker's ability from that of his effort on output. In response, the worker works harder to improve the market's perception of his ability. In comparison, in our model, the labor market cannot distinguish the impact of the worker's ability from that of tuition on education level. Thus, the school has an incentive to secretly cut tuition, thereby improving the market's perception and stimulating demand. In Chan, Li, and Suen (2007), a school has an incentive to inflate grades to improve the market's perception of its students. They show that grade inflation features strategic complements when the qualities of students are correlated across schools. In contrast to their model, our model incorporates screening in addition to signaling, as the school cannot observe its students' abilities.

Finally, our paper relates closely to the literature on intermediate price transparency. Inderst and Ottaviani (2012) shows how product providers compete through commissions paid to consumer advisers. Commissions bias advice; thus, an increase in a firm's commission reduces consumers' willingness to pay if they observe the commission. Analogously, in our model, cheaper tuition reduces the signaling value of education, and thus, tuition cuts are less effective at stimulating demand than they would be otherwise when employers observe tuition. In Janssen and Shelegia (2015), a manufacturer chooses a wholesale price, retailers choose retail prices, and consumers search for the best deal. They argue that retailers are less sensitive to wholesale price changes when consumers do not observe the price than otherwise, as uninformed consumers are more likely to keep searching when the retail price raises. By contrast, in our model the worker is more sensitive to tuition changes when employers do not observe the tuition scheme than otherwise, as uninformed employers will have better (worse) beliefs over the worker's ability if they observe a higher (lower) education level.

2 The Model

Players and actions. There is a single school (*principal*), a worker (*agent*) and multiple identical and competing firms, also referred to as *the labor market*. At the beginning of the game, the school chooses a tuition scheme $T(z) : \mathbb{R}_+ \to \mathbb{R}_+$, where z stands for education level and $T(z)$ is the tuition at z. Subsequently, the worker decides how much education to purchase from the school based on the tuition scheme. For simplicity, we do not explicitly model firms' actions; rather, we directly assume that they offer the worker a wage equal to his expected productivity (see below).

The worker's productivity depends on his ability (*type*) θ and his education choice z. Specifically, θ is a random variable, which distributes over the interval $[\underline{\theta}, \bar{\theta}]$, according to a distribution function $F(\theta)$ with a positive density function $f(\theta)$. Denote by $Q(z, \theta)$ the productivity of a type-θ worker having education level z. We assume that $Q(z, \theta)$ is twice differentiable and increasing in both arguments. Formally, $Q_z(z, \theta)$, $Q_\theta(z, \theta) > 0$ if $z > 0$. We also assume that a worker with no education has zero productivity irrespective of his ability; that is, $Q(0, \theta) \equiv 0$. We consider this assumption realistic since many jobs require a minimal education level. For example, a lawyer candidate must graduate from a law school, and medical school education is prerequisite for being a licensed practitioner of medicine. In the Appendix, as a supplementary exercise, we present the analysis for the case in which education is unproductive.

Information. The worker's education level is publicly observed. However, neither the school nor the labor market observes the worker's ability, but both know its distribution. In this paper, we mainly study two variants of the model: in the *observed* case, the tuition scheme is observed by the labor market; in the *unobserved* case, it is unobserved by the labor market. In each case, the labor market announces and commits to a wage schedule $W(z) : \mathbb{R}_+ \to \mathbb{R}_+$, based on the available information.

Payoffs. We normalize the school's marginal costs of educating to zero and abstract from fixed costs. Suppose that the school chooses some tuition scheme $T(z)$; then, let $z(\theta; T)$ be the education level chosen by a type-θ worker under $T(z)$. Given the tuition scheme $T(z)$ and the wage schedule $W(z)$, a type-θ worker who chooses education level z has utility:

$$U(z, \theta) \equiv W(z) - T(z) - C(z, \theta),$$

where $C(z, \theta)$ is the worker's cost of effort for education. We assume that $C(z, \theta)$ is twice differentiable, increasing and strictly convex in z, and unbounded: $C_z(z, \theta) > 0$ if $z > 0$, and $C_{zz}(z, \theta) > k$ for some $k > 0$. Moreover, the standard *single-crossing property* holds: $C_{z\theta}(z, \theta) < 0$ if $z > 0$. This condition captures the feature that a higher-ability worker has lower marginal effort costs than a lower-ability worker. We also normalize $C(0, \theta)$ to 0 for all $\theta \in [\underline{\theta}, \bar{\theta}]$. This implies that, combined with $C_z(z, \theta) > 0$ and $C_{z\theta}(z, \theta) < 0$ if $z > 0$, $C_\theta(z, \theta) < 0$ if and only if $z > 0$. Finally, we assume that the worker can obtain a zero-utility outside option by acquiring no education and not entering the labor market.

First-best benchmark. Define $S(z, \theta)$ as the social surplus function, i.e.,

$$S(z, \theta) = Q(z, \theta) - C(z, \theta).$$

Assume that $S(z, \theta)$ is strictly quasiconcave in z and has a unique maximizer $z^{fb}(\theta) \geq 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Then, the first-order condition implies that

$$S_z(z^{fb}(\theta), \theta) = Q_z(z^{fb}(\theta), \theta) - C_z(z^{fb}(\theta), \theta) = 0. \tag{2.1}$$

To ensure that $z^{fb}(\theta)$ is increasing, we assume further that $S_{z\theta}(z, \theta) > 0$ if $z > 0$. Then, the monotonicity holds according to Milgrom and Shannon (1994, Theorem 4). It is also readily confirmed that $S(z^{fb}(\theta), \theta)$ is increasing in θ.

Equilibrium. We use *perfect Bayesian equilibrium* as the solution concept throughout the paper. In the observed case, an equilibrium consists of the school's tuition scheme $T^o(z)$ and conditional on any tuition scheme $T(z)$, the worker's education function $z^o(\theta; T)$ and the labor market's wage schedule $W^o(z; T)$, such that

(i) For each $T(z)$, the following holds: (a) given $W^o(z; T)$, $z^o(\theta; T)$ maximizes $U(z, \theta)$; (b) $W^o(z; T) = \mathbb{E}[Q(z, \theta) | z^o(\theta; T)]$ such that the labor market's posterior belief about the worker's ability, or simply *the market belief*, is updated using Bayes' rule.

(ii) Given $z^o(\theta; T)$, $T^o(z)$ maximizes the school's expected profit, i.e.,

$$T^o(z) \in \underset{T(z)}{argmax} \int_{\underline{\theta}}^{\bar{\theta}} T(z^o(\theta; T)) dF(\theta).$$

In the unobserved case, the market's inference is independent of the actual tuition scheme but is conditional on a *conjectured* scheme; in equilibrium, the conjecture is correct. In this case, an equilibrium consists of a tuition scheme $T^u(z)$ and a wage schedule $W^u(z)$ (more precisely, $W^u(z;T^u)$), and conditional on any $T(z)$, an education function $z^u(\theta;T)$, such that

(*i*) Given $W^u(z)$, for each $T(z)$, $z^u(\theta;T)$ maximizes $U(z,\theta)$; $W^u(z) = \mathbb{E}[Q(z,\theta)|z^u(\theta;T^u)]$ such that the market belief is updated using Bayes' rule.

(*ii*) Given $z^u(\theta;T)$, $T^u(z)$ maximizes the school's expected profit, i.e.,

$$T^u(z) \in \underset{T(z)}{argmax} \int_{\underline{\theta}}^{\bar{\theta}} T(z^u(\theta;T))dF(\theta).$$

Note that the equilibrium conditions have one important difference between the observed and unobserved case: in the unobserved case, the market belief needs to be correct only on the equilibrium path, whereas in the observed case, the market belief has to be correct following every tuition scheme that is chosen by the school.

Equilibrium selection. For both the observed and unobserved case, while there possibly exist multiple equilibria, we focus on the *school-optimal separating equilibrium*, that is, the equilibrium that yields the highest payoff for the school, provided that on the equilibrium path, $z(\theta)$ is one-to-one if $z(\theta) > 0$.[3] To ensure that a separating equilibrium indeed exists, we impose an assumption on the cost function $C(z,\theta)$ and the distribution function $F(\theta)$.

Assumption 2.1. *$C_{z\theta\theta}(z,\theta) \geq 0$ and $F(\theta)$ has a non-decreasing hazard rate.*

The reason that we select the school-optimal separating equilibrium is because in Spence's model, the unique equilibrium that survives the D1 refinement (Banks and Sobel 1987) is the *least-cost separating equilibrium* (Riley 1979) in which $z(\theta)$ is one-to-one and the lowest type $\underline{\theta}$ chooses the first-best $z^{fb}(\underline{\theta})$.[4] In the school-optimal separating equilibrium, given the equilibrium tuition scheme, the continuation game constitutes the least-cost separating equilibrium, thereby allowing us to compare the associated equilibrium predictions with that of Spence's model. Moreover, in a discrete-type version of the model, a pooling equilibrium, in which all participating types choose an identical education level, does not exist in either

[3] We do not impose any restriction on $z(\theta)$ off the equilibrium path.

[4] Since $\underline{\theta}$ is the worst market belief, the lowest type does not fear further punishment from deviating to its full-information optimal education level. This initial condition leads to the separating equilibrium in which the worker obtains the least education.

case whenever the proportion of the highest type is sufficiently large.[5] Hence, we consider the school-optimal separating equilibrium a reasonable equilibrium to study. In Sections 4 and 5, we will discuss equilibrium selection in greater detail.

Finally, given the equilibrium selection rule, one can easily conclude that the school has a higher equilibrium payoff in the observed case than in the unobserved case, as it can secure a weakly higher expected profit in the observed case by maintaining $T^u(z)$. In what follows, we apply the revelation principle to simplify the equilibrium characterization for both cases.

2.1 Communication Mechanisms

Appealing to the revelation principle, we consider communication mechanisms between the school and worker in both the observed and unobserved case. It is without loss of generality to adjust the timing as follows. First, the school offers a contract $\langle z(\theta), T(z)\rangle$ to the worker. Then, the labor market publishes a wage schedule $W(z)$ based on the information available: in the observed case, it observes the contract; in the unobserved case, it does not. Finally, the worker reports his type to only the school. Reporting a type $\hat{\theta}$, the worker obtains education level $z(\hat{\theta})$, pays tuition $T(z(\hat{\theta}))$ and then receives wage $W(z(\hat{\theta}))$.

Worker's problem. In both cases, given a contract $\langle z(\theta), T(z)\rangle$ and the associated wage schedule $W(z)$, a type-θ worker chooses a report $\hat{\theta}$ to maximize his utility

$$U(\hat{\theta}, \theta) = W(z(\hat{\theta})) - T(z(\hat{\theta})) - C(z(\hat{\theta}), \theta).$$

The mechanism $\{\langle z(\theta), T(z)\rangle, W(z)\}$ is *incentive compatible* if the worker is willing to truthfully report his type and is *individually rational* if the worker obtains a non-negative utility level. A type-θ worker's equilibrium payoff is represented by $U(\theta) \equiv U(\theta, \theta)$.

School's problem. In the observed case, the school chooses a contract to maximize its expected profit subject to incentive compatibility, individual rationality, and the market belief being correct. In the unobserved case, since the market's inference is independent of the school's choice, given the wage schedule, the school chooses a contract to maximize its expected profit subject to incentive compatibility and individual rationality.

[5]Suppose that, in either the observed or unobserved case, a pooling equilibrium exists such that all participating types choose the same education; then, the equilibrium wage is a constant which equals the average productivity, and the equilibrium tuition is a fixed fee and makes the lowest participating type just indifferent. But whenever the proportion of the highest type is sufficiently large, it is optimal for the school to serve only the highest type and exclude all lower types, leading to a contradiction.

Preliminaries. In both cases, an allocation $\langle z(\theta), U(\theta)\rangle$ is *implementable* if it is incentive compatible and individually rational. Appealing to Mas-Colell, Whinston, and Green (1995, Proposition 23.D.2), we characterize all implementable allocations by the following lemma.

Lemma 2.1. *In both cases, an allocation $\langle z(\theta), U(\theta)\rangle$ is implementable if and only if*

(i) $z(\theta)$ is non-decreasing.

(ii) Define $\theta_0 \equiv \inf\{\theta | z(\theta) > 0\}$; then, for $\theta > \theta_0$,

$$U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} -C_\theta(z(s), s)ds$$

subject to $U(\theta_0) \geq 0$.

By Lemma 2.1, we can rewrite the school's problem for both cases. Note that incentive compatibility means that $T(z(\theta)) = W(z(\theta)) - C(z(\theta), \theta) - U(\theta)$ and that $U(\theta_0)$ is optimally set to 0. Substituting and integrating by parts, the school's problem can be stated as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ W(z(\theta)) - C(z(\theta), \theta) + \frac{1 - F(\theta)}{f(\theta)} C_\theta(z(\theta), \theta) \right\} dF(\theta) \tag{2.2}$$

subject to $z(\theta)$ being non-decreasing.

In the observed case, correctness of the market belief means that $W(z) = \mathbb{E}[Q(z, \theta)|z(\theta)]$ for any implementable allocation $z(\theta)$ that the school chooses. Then, from the law of total expectation, Program (2.2) is equivalent to

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ S(z(\theta), \theta) + \frac{1 - F(\theta)}{f(\theta)} C_\theta(z(\theta), \theta) \right\} dF(\theta) \tag{2.3}$$

subject to $z(\theta)$ being non-decreasing. Intuitively, because firms break even in expectation, the school maximizes the expected difference between social surplus and consumer surplus, as in Mussa and Rosen (1978) or Maskin and Riley (1984). It suffices to solve Program (2.3) for the equilibrium characterization of the observed case. If the solution $z^o(\theta)$ is increasing over $[\theta_0, \bar{\theta}]$, then we obtain the school-optimal separating equilibrium.

In the unobserved case, without loss of generality, the school chooses an allocation $z(\theta)$, while simultaneously, the labor market chooses a wage schedule $W(z)$. Then, the equilibrium conditions can be simplified as follows: (i) given $W^u(z)$, $z^u(\theta)$ solves the school's problem in (2.2); (ii) $W^u(z) = \mathbb{E}[Q(z, \theta)|z^u(\theta)]$ such that the market belief is updated using Bayes' rule. In the case of multiple equilibria, we select the school-optimal separating equilibrium.

2.2 A Simple Example with Two Types

To develop simple intuitions for our general results, we establish a numerical example with binary types. We assume that $\theta \in \{\theta_L, \theta_H\}$ with $0 < \theta_L < \theta_H$ and that each type is realized with equal probability. We also assume that $Q(z, \theta) = \theta z$ and $C(z, \theta) = z^2/(2\theta)$. It is readily confirmed that Assumption 2.1 holds in this example.

We first study the observed case. Suppose that the contract $\{(z_L^o, T_L^o), (z_H^o, T_H^o)\}$ solves the school's problem and that the associated wage schedule is given by $\{W_L^o, W_H^o\}$. Then, incentive compatibility and individual rationality imply that

$$C(z_L^o, \theta_L) - C(z_L^o, \theta_H) \leq U(\theta_H) - U(\theta_L) \leq C(z_H^o, \theta_L) - C(z_H^o, \theta_H), \tag{IC}$$

$$U(\theta_i) = W_i^o - T_i^o - C(z_i^o, \theta_i) \geq 0, \;\; i = L, H. \tag{IR}$$

Note that $T_i^o = W_i^o - C(z_i^o, \theta_i) - U(\theta_i)$, $i = L, H$; thus, the school's expected profit equals

$$\Pi^o = 0.5\,(T_L^o + T_H^o) = 0.5\left[W_L^o + W_H^o - C(z_L^o, \theta_L) - C(z_H^o, \theta_H) - U(\theta_L) - U(\theta_H)\right].$$

As is standard in the literature, both the downward IC and the low type's IR constraints are binding. Moreover, a correct market belief means that $W_L^o + W_H^o = \theta_L z_L^o + \theta_H z_H^o$ regardless of whether it is separating ($z_L^o \neq z_H^o$). Substituting these results and the model assumptions into Π^o, we write the school's problem as follows:

$$\max_{z_L, z_H} \; \theta_L z_L - \frac{z_L^2}{2\theta_L} - \frac{(\theta_H - \theta_L) z_L^2}{2\theta_L \theta_H} + \theta_H z_H - \frac{z_H^2}{2\theta_H} \quad s.t. \quad z_H \geq z_L.$$

Then, the first-order conditions imply that

$$z_L^o = \frac{\theta_H \theta_L^2}{2\theta_H - \theta_L} < \theta_L^2 = z_L^{fb} \;\; \text{and} \;\; z_H^o = \theta_H^2 = z_H^{fb}.$$

Since $z_L^o < z_H^o$, we obtain the school-optimal separating equilibrium. In this equilibrium, the low type chooses less education than the first-best, while the high type chooses exactly the first-best. Intuitively, the high type benefits from his cost advantage over the low type, as indicated by the downward IC constraint, and thus, he extracts an *information rent* that is increasing in the low type's education level. This induces the school to under-supply education to the low type. Since there are only two types, there is no distortion of the high type's education level. From preliminary calculations, one can completely characterize the equilibrium outcome. In particular, the school's equilibrium payoff equals

$$\Pi^o = 0.5\,(T_L^o + T_H^o) = \frac{\theta_H \theta_L^3}{8\theta_H - 4\theta_L} + \frac{\theta_H^3}{4}.$$

We then consider the unobserved case. Suppose that the labor market believes naively that the school's contract is the same as that in the observed case and thus offers the same wage schedule. Will the school retain the same contract? It depends. To see why, consider an alternative contract $\{(z', T')\}$ such that $z' = z^o_H$ and $T' = W^o_H - C(z^o_H, \theta_L) < T^o_H$. That is, the school only offers the high education level from the observed case and reduces tuition to the level that also attracts the low type. Thus, the school's new expected profit equals

$$\Pi' = T' = \theta_H^3 - \frac{\theta_H^4}{2\theta_L}.$$

One can show that if θ_L and θ_H are close enough to one another, e.g., $\theta_L = 1$ and $\theta_H = 1.1$, then this deviation is indeed profitable ($\Pi' \approx 0.60$ while $\Pi^o \approx 0.56$). The idea is that with the wage schedule being fixed, the school secretly cuts its prices to gain market share; if the labor market could observe the tuition, such price cuts would undermine the signaling value of education and thus would not be profitable. If the gap between types is small enough, then the increase in quantity dominates the reduction in price, making the deviation profitable. In contrast, if the gap is relatively big, e.g., $\theta_L = 1$ and $\theta_H = 2$ (in this case, $\Pi' = 0$ while $\Pi^o = 13/6$), and any off-equilibrium-path education is believed to be chosen by the low type, then the equilibrium of the observed case can also be sustained in the unobserved case. This is because the high education level is relatively high, and thus, the school finds it unprofitable to induce the low type to imitate the high type by secretly cutting the price.

Then, what is the equilibrium of the unobserved case if the aforementioned deviation is profitable? Here, we characterize the school-optimal separating equilibrium without proof. First, the offer to the low type (z^u_L, T^u_L) is the same as (z^o_L, T^o_L). Second, the high education level z^u_H satisfies the school's incentive compatibility constraint as follows:

$$\theta_L z^u_L - \frac{z^{u2}_L}{2\theta_L} - \frac{(\theta_H - \theta_L) z^{u2}_L}{2\theta_L \theta_H} = \theta_H z^u_H - \frac{z^{u\,2}_H}{2\theta_L} - \frac{(\theta_H - \theta_L) z^{u\,2}_H}{2\theta_L \theta_H} \quad s.t. \;\; z^u_H \geq z^u_L. \tag{SIC}$$

This constraint indicates that the school weakly prefers truthfully revealing the worker's type to inducing the low type to imitate the high type. Third, W^u equals $Q(z, \theta_H)$ if $z = z^u_H$ and equals $Q(z, \theta_L)$ otherwise. Finally, T^u_H is derived by substituting z^u_H into $S(z, \theta_H) - U(\theta_H)$, where $U(\theta_H) = C(z^u_L, \theta_L) - C(z^u_L, \theta_H)$, as the downward IC constraint is binding.

This equilibrium outcome reveals our second heuristic result. In the unobserved case, the high type selects more education than in the observed case (note that the low type's situation does not change). One can derive this result from the SIC constraint. For example, if $\theta_L = 1$ and $\theta_H = 1.1$, then $z^u_H \approx 1.43$ while $z^o_H = z^{fb}_H = 1.21$. The underlying intuition is that in the unobserved case, the high education level must be excessively high such that the school finds it unprofitable to induce the low type to imitate the high type. Since in both cases the downward IC constraint is binding and the low type's education level is the same, the high type's utility does not change between the two cases. This implies that the high education level must be cheaper in the unobserved case, and thus, the school has a lower profit than in the observed case. For example, if $\theta_L = 1$ and $\theta_H = 1.1$, then $T_H^u \approx 0.61$ while $T_H^o = 0.63$. The reason is that the high type selects the efficient quantity in the observed case but selects more than the efficient quantity in the other, and thus, less social surplus is generated in the unobserved case.[6] To make the high type no worse off, the school must charge a lower price for the high education level. Intuitively, since education is inflated ($z_H^u > z_H^o$), the signaling value of the high education is diluted. This means that the worker has a lower willingness to pay, and thus, the school achieves lower profits in the unobserved case.

3 Job Market Signaling without Tuition

We now return to the general model. As a reference point, we revisit Spence's signaling game in which tuition is fixed at zero. One could interpret such a benchmark as the case in which schools are competitive and thus choose tuition equal to the marginal cost. In this case, an equilibrium consists of an education function $z^s(\theta)$ and a wage schedule $W^s(\theta)$, such that (i) given $W^s(\theta)$, $z^s(\theta)$ maximizes $U(z,\theta)$; (ii) $W^s(\theta) = \mathbb{E}[Q(z,\theta)|z^s(\theta)]$ with the market belief updated using Bayes' rule. We focus on the least-cost separating equilibrium, in which $z^s(\theta)$ is one-to-one and the lowest type $\underline{\theta}$ chooses the first-best $z^{fb}(\underline{\theta})$. In the following, we apply the general results of Mailath (1987) to our specific setting to characterize the equilibrium.

Proposition 3.1. *The least-cost separating equilibrium exists, such that*

(i) $z^s(\underline{\theta}) = z^{fb}(\underline{\theta})$; $z^s(\theta)$ *satisfies the first-order condition*

$$Q_z(z^s(\theta),\theta) + Q_\theta(z^s(\theta),\theta)\cdot\theta^{s\prime}(z^s(\theta)) - C_z(z^s(\theta),\theta) = 0, \tag{3.1}$$

where $\theta^s(z)$ *is the inverse function of* $z^s(\theta)$*, being differentiable on* $[\underline{\theta},\bar{\theta}]$*.*

(ii) $z^s(\theta)$ *is increasing over* $[\underline{\theta},\bar{\theta}]$*, and thus,* $W^s(z^s(\theta)) = Q(z^s(\theta),\theta)$ *for all* $\theta \in [\underline{\theta},\bar{\theta}]$*.*

Note that the first two terms on the left-hand side (LHS) of (3.1) are the total derivative of $W^s(z)$. In particular, the second term is non-negative given the monotonicity of $z^s(\theta)$.

[6] This implies that social welfare is lower in the unobserved case in the two-type model. In the general model, however, the welfare comparison between the two cases is ambiguous, as we will show in Section 5.

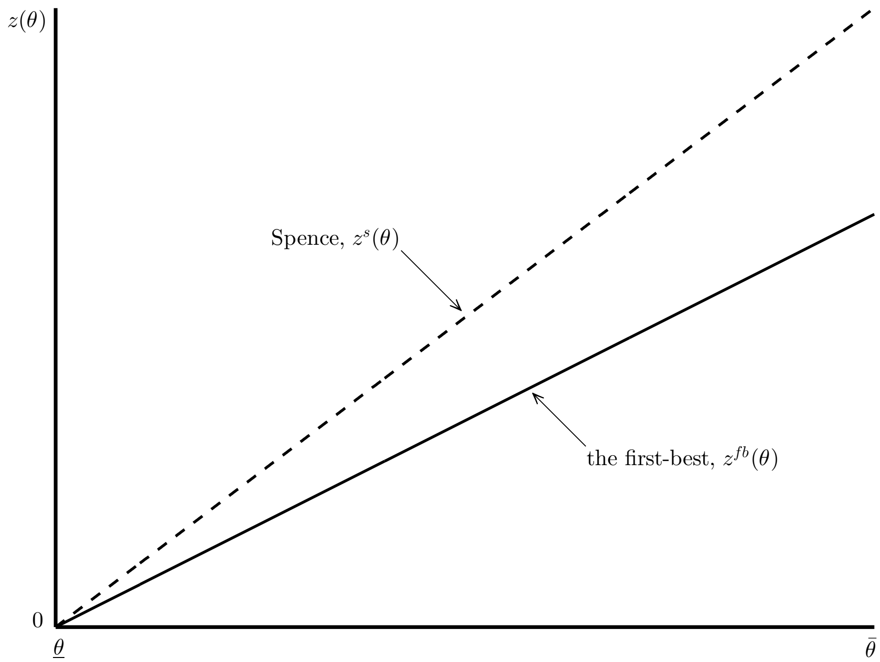

Figure 1: **The Signaling Effect.** This figure compares $z^s(\theta)$ with $z^{fb}(\theta)$ over $[\underline{\theta}, \bar{\theta}]$. This figure assumes that $Q(z, \theta) = \theta z + z$, $C(z, \theta) = z^2 + z - \theta z$, and $\theta \sim U[0, 1]$. In this example, $z^{fb}(\theta) = \theta$ and $z^s(\theta) = \frac{3}{2}\theta$.

Since $S(z, \theta)$ is strictly quasiconcave, comparing (3.1) with (2.1) implies that $z^s(\theta) \geq z^{fb}(\theta)$ for all $\theta \geq \underline{\theta}$, with equality holding at $\underline{\theta}$ only. This comparison is illustrated in Figure 1. We now summarize this result in the following corollary.

Corollary 3.1. *In Spence's signaling game, the worker chooses more education than the first-best. Specifically,* $z^s(\theta) \geq z^{fb}(\theta)$ *on* $[\underline{\theta}, \bar{\theta}]$*, with strict inequality for* $\theta > \underline{\theta}$.

Corollary 3.1 indicates that the worker's signaling activity leads to over-education. The intuition is well-understood. Under complete information, the marginal benefit of education is its marginal contribution to human capital. In contrast, when ability is privately known, in addition to the human capital effect, there is a *signaling effect*; that is, a higher education level makes the labor market regard the worker as having higher ability. Thus, the marginal benefit of education is higher than under complete information. Since the marginal cost is the same, education is over-invested in when the worker's ability is private information.

4 Labor Market Observes Tuition

Starting with this section, we take the school's strategic behavior into account. Here, we consider the case in which the labor market observes the tuition scheme. From Section 2.1, it suffices to solve the school's problem in (2.3) for the equilibrium characterization. It is heuristic to interpret the integrand in (2.3) as the school's marginal profit in the observed case. Define

$$MP^o(z,\theta) \equiv S(z,\theta) + \frac{1-F(\theta)}{f(\theta)} C_\theta(z,\theta).$$

As is standard in the literature, we solve the school's problem with the monotonicity constraint relaxed. This is equivalent to pointwise optimization for $MP^o(z,\theta)$. Inspired by Martimort and Stole (2009), we say that the school's marginal profit in the observed case is *regular* if $MP^o(z,\theta)$ is strictly quasiconcave in z and $MP^o_z(z,\theta)$ is increasing in θ. Given Assumption 2.1, regularity holds. Thus, $MP^o(z,\theta)$ has a unique maximizer $z^*(\theta)$, which is increasing. Note that $z^*(\theta)$ might be negative for some region of θ; as such, we set $z(\theta)$ to 0 instead of $z^*(\theta)$. Hence, $MP^o(z(\theta),\theta)$ is non-decreasing in θ and is non-negative. The cutoff type θ^o_0 is thus either the maximal root of $MP^o(z(\theta),\theta) = 0$ if it exists, or $\underline{\theta}$ otherwise. In summary, the optimal allocation $z^o(\theta)$ is given by

$$z^o(\theta) = \begin{cases} z^*(\theta) & \text{if } \theta \geq \theta^o_0 \\ 0 & \text{otherwise.} \end{cases} \tag{4.1}$$

To complete the characterization, back out $z^o(\theta)$'s inverse function $\theta^o(z)$ on $[\theta^o_0, \bar{\theta}]$ given its monotonicity. Plugging $\theta^o(z)$ into $Q(z,\theta)$ yields the wage schedule $W^o(z)$ on $[z^o(\theta^o_0), z^o(\bar{\theta})]$. Finally, the tuition scheme $T^o(z)$ on $[z^o(\theta^o_0), z^o(\bar{\theta})]$ is given by

$$T^o(z^o(\theta)) = S(z^o(\theta),\theta) - U(\theta) = S(z^o(\theta),\theta) + \int_{\theta^o_0}^{\theta} C_\theta(z^o(s),s)ds. \tag{4.2}$$

For the off-path education levels, we assume without loss that the school sets exorbitantly high prices such that no type is willing to deviate to there in any case. Then, given $T^o(z)$, the school-optimal separating equilibrium is also the least-cost separating equilibrium in the sense that the cutoff type chooses his full-information optimal quantity under the total cost function $T^o(z) + C(z,\theta)$. Moreover, since $z^o(\theta)$ coincides with the unconstrained optimizer $z^*(\theta)$ on path, this equilibrium is further *the school-optimal equilibrium*. We now summarize the equilibrium outcome of the observed case in the proposition below.

Proposition 4.1. *Given Assumption 2.1, the school-optimal separating equilibrium exists. On the equilibrium path, the education function $z^o(\theta)$ is given by (4.1); the tuition scheme $T^o(z)$ is given by (4.2); and the wage schedule $W^o(z)$ equals $Q(z,\theta^o(z))$.*

Note that $MP^o_z(z,\theta)$ is less than $S_z(z,\theta)$, holding weakly on the boundary. Consequently, regularity implies that $z^o(\theta) \leq z^{fb}(\theta)$ on $[\theta^o_0, \bar{\theta}]$, with equality holding at $\bar{\theta}$ only. Particularly, if $\theta^o_0 > \underline{\theta}$, then $z^o(\theta) = 0$ for all $\theta \in [\underline{\theta}, \theta^o_0)$. To summarize, we have the following corollary:

Corollary 4.1. *In the observed case, the worker chooses less education than the first-best. Specifically, $z^o(\theta) \le z^{fb}(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality on $(\underline{\theta}, \bar{\theta})$.*

Corollary 4.1 states that when the labor market observes the tuition scheme, education is under-supplied. This result stands in stark contrast to that of Spence's model. The altered equilibrium prediction results from the school's screening activity. Specifically, having a cost advantage in education, a higher-ability worker can secure higher utility than a lower-ability worker by choosing the same education as the latter. Therefore, to incentivize truth-telling, the school has to leave information rents to the worker. This means that the marginal profit of education is less than the social surplus generated; therefore, the school under-supplies education. In particular, an interval of types at the low end of the domain will be excluded from education if it is too costly to serve them.

Remark. Suppose that education is a pure signal (i.e., $Q(z,\theta) \equiv Q(\theta)$) as in Spence (1973); we show in Appendix A.3 that the school-optimal separating equilibrium yields a *virtually* socially optimal outcome, such that $z^o(\theta)$ is arbitrarily close to $z^{fb}(\theta) \equiv 0$. Specifically, the school allocates increasing and infinitesimal education to different types, with the lowest type having no education. Thus, the school's profit is arbitrarily close to the first-best social welfare minus the lowest type's utility $Q(\underline{\theta})$. To interpret, since education is unproductive, the school provides little and different education in the form of different types of degrees to separate types; a worker without any degree is regarded as having the lowest ability.

4.1 Screening vs Signaling

While the equilibrium prediction for the observed case is due to the mechanism of monopoly screening, our model also contains signaling. Note that given the tuition scheme $T^o(z)$, the subgame is indeed Spence's signaling game as if the worker had a cost function in the form of $T^o(z) + C(z,\theta)$. From the same argument as in Corollary 3.1, the education levels in the observed case are distorted, due to signaling, above the "efficient" level with respect to the total cost of education. This fact reveals that the equilibrium outcome of the observed case results from the interaction between screening and signaling.

Corollary 4.1 implies that when both screening and signaling are present and exert the opposite effects—screening induces under-education, but signaling induces over-education—screening outweighs signaling. This is because as a Stackelberg leader, the school internalizes the worker's signaling incentive when screening his type. To see this, note that

$$T^{o\prime}(z) = W^{o\prime}(z) - C_z(z, \theta^o(z)) = \frac{d}{dz}\left[Q(z, \theta^o(z))\right] - C_z(z, \theta^o(z)).$$

Substituting this equation into the first-order condition of $MP^o(z,\theta)$, we have

$$T^{o\prime}(z) = Q_\theta(z, \theta^o(z)) \cdot \theta^{o\prime}(z) + \frac{1 - F(\theta^o(z))}{f(\theta^o(z))}\left[-C_{z\theta}(z, \theta^o(z))\right]. \tag{4.3}$$

On the right-hand side (RHS) of (4.3), the first term captures the signaling effect, and the second term is the marginal information rent extracted by the worker. Note that signaling induces over-education, which reduces the school's profit in two ways: on one hand, it lowers total surplus; on the other hand, it provides the worker with more information rents. Thus, the optimal tuition scheme must undo these two effects, as indicated by (4.3). In contrast, if the school were a welfare-maximizing social planner, it would only undo the signaling effect by levying Pigovian taxes (Spence 1974). Denote by $T^{fb}(z)$ the welfare-maximizing tax on education. The marginal tax is equal to the signaling effect at the first-best, i.e.,

$$T^{fb\prime}(z) = Q_\theta(z, \theta^{fb}(z)) \cdot \theta^{fb\prime}(z), \tag{4.4}$$

where $\theta^{fb}(z)$ is the inverse function of $z^{fb}(\theta)$.[7] Since the second term on the RHS of (4.3) is positive, comparing (4.3) with (4.4) indicates that the profit-maximizing tax on education "over-taxes" signaling activity and thus leads to under-education.

To see how signaling makes a difference, consider the situation in which the labor market also observes the worker's ability without changing any other element of the model. In this case, the wage equals the actual productivity, and signaling is eliminated. This means that the worker's intrinsic value for education is the social surplus $S(z,\theta)$. Since $S_\theta = Q_\theta - C_\theta > 0$, a higher type can be seen as a higher-value buyer of education. Thus, the school has the same monopoly screening problem as in Mussa and Rosen (1978). Specifically, the school chooses a contract $\langle z(\theta), T(z)\rangle$ to maximize its expected profit subject to incentive compatibility and individual rationality. Analogous to Lemma 2.1, an allocation $\langle z(\theta), U(\theta)\rangle$ is implementable if and only if (*i*) $z(\theta)$ is non-decreasing; (*ii*) $U(\theta_0) \geq 0$ and for $\theta > \theta_0$,

$$U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} S_\theta(z(s), s)ds.$$

Thus, the school's problem in such Mussa and Rosen's screening game can be stated as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ S(z(\theta), \theta) - \frac{1 - F(\theta)}{f(\theta)} S_\theta(z(\theta), \theta) \right\} dF(\theta)$$

subject to $z(\theta)$ being non-decreasing.

[7] Since the lowest type $\underline{\theta}$ chooses the first-best $z^{fb}(\underline{\theta})$ in equilibrium, he should be exempt from such tax; that is, $T^{fb}(z^{fb}(\underline{\theta})) = 0$. Then, directly integrating (4.4) yields the welfare-maximizing tax scheme $T^{fb}(z)$.

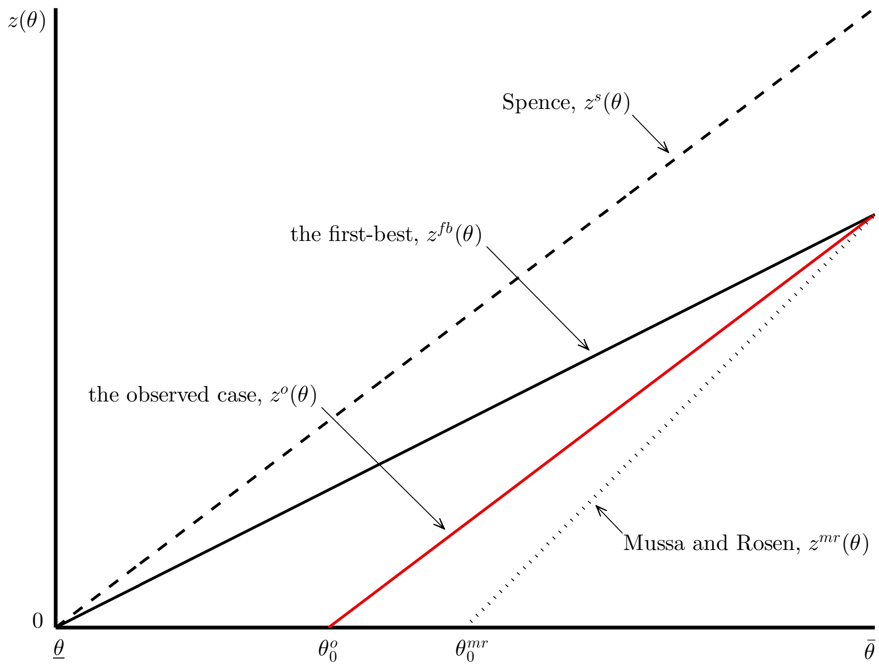

Figure 2: **Screening vs Signaling.** This figure compares $z^{mr}(\theta)$ with $z^o(\theta)$ over $[\underline{\theta}, \bar{\theta}]$ based on Figure 1. This figure assumes the same numerical example as Figure 1, such that $z^o(\theta) = \frac{3\theta-1}{2}$ and $z^{mr}(\theta) = 2\theta - 1$.

Define analogously the school's marginal profit as

$$MP^{mr}(z,\theta) \equiv S(z,\theta) - \frac{1-F(\theta)}{f(\theta)} S_\theta(z,\theta).$$

Similarly, we say that $MP^{mr}(z,\theta)$ is regular if it is strictly quasiconcave in z and $MP_z^{mr}(z,\theta)$ is increasing in θ.[8] Denote by $z^{mr}(\theta)$ and θ_0^{mr} the optimal allocation and the cutoff type in Mussa and Rosen's model, respectively. Suppose that $MP^{mr}(z,\theta)$ is regular, then one can characterize $z^{mr}(\theta)$ and θ_0^{mr} analogously to the observed case.

We shall examine how the allocation in Mussa and Rosen's model differs from that in the observed case. On the extensive margin, because $S_\theta > -C_\theta$, $MP^{mr}(z,\theta) \leq MP^o(z,\theta)$, with strict inequality for $\theta < \bar{\theta}$. Hence, if $\theta_0^o > \underline{\theta}$, then $\theta_0^{mr} > \theta_0^o$; that is, more types are excluded in Mussa and Rosen's model. On the intensive margin, if $Q_{z\theta} > 0$ on $[0, z^{fb}(\bar{\theta})]$,[9] then $z^{mr}(\theta) \leq z^o(\theta)$, with strict inequality on $[\theta_0^o, \bar{\theta})$, meaning that under-education is more serious in Mussa and Rosen's model. These findings are illustrated in Figure 2.

For welfare comparison, note that education is already under-supplied in the observed case, yet the downward distortion is larger in Mussa and Rosen's model; thus, the observed

[8] Given Assumption 2.1, $MP^{mr}(z,\theta)$ is regular if $Q_{z\theta\theta} \leq 0$.

[9] This condition is not restrictive; indeed, given that $Q_\theta(z,\theta) > 0$ and $Q(0,\theta) \equiv 0$, we have $Q_{z\theta}(z,\theta) > 0$ on $[0, \bar{z}]$ for some $\bar{z} > 0$. Given this condition, $MP_z^{mr}(z,\theta) < MP_z^o(z,\theta)$ on $[0, z^{fb}(\bar{\theta})]$ for $\theta < \bar{\theta}$.

case has higher social welfare. Moreover, since $MP^{mr}(z^{mr}(\theta),\theta) \leq MP^o(z^o(\theta),\theta)$ with strict inequality on $[\theta_0^o,\bar{\theta})$ and $\theta_0^{mr} \geq \theta_0^o$, it is readily confirmed that the school's expected profit is also higher in the observed case. In summary, we have the following proposition:

Proposition 4.2. *If both $MP^o(z,\theta)$ and $MP^{mr}(z,\theta)$ are regular, and $Q_{z\theta} > 0$ on $[0, z^{fb}(\bar{\theta})]$, then under-education is greater when signaling is eliminated. Specifically, $z^{mr}(\theta) \leq z^o(\theta)$, with strict inequality on $[\theta_0^o,\bar{\theta})$; if $\theta_0^o > \underline{\theta}$, then $\theta_0^{mr} > \theta_0^o > \underline{\theta}$. Consequently, social welfare and the school's expected profit are strictly higher when signaling is present than otherwise.*

Proposition 4.2 indicates that signaling can mitigate the downward distortion caused by screening. Intuitively, when the labor market observes the worker's ability, if a higher type imitates a lower type by choosing the same education, he not only has a lower total cost than the latter but also obtains a higher wage due to his higher productivity. In contrast, when the labor market does not observe the worker's ability, the higher type can no longer directly reap the benefit from higher productivity, and thus, he acquires more education to signal his ability. The signaling incentive reduces the worker's willingness to imitate lower types. Therefore, the school leaves lower information rents to the worker when signaling is present, as we have the following inequality,

$$\underbrace{\frac{1-F(\theta)}{f(\theta)}[-C_\theta(z,\theta)]}_{\text{information rents with signaling}} \leq \underbrace{\frac{1-F(\theta)}{f(\theta)}S_\theta(z,\theta)}_{\text{information rents without signaling}}$$

which holds with equality at $\bar{\theta}$ only. Consequently, the screening distortion is mitigated.

4.2 Signaling Intensity, Market Structure and Welfare

Knowing that signaling can mitigate the downward distortion due to screening, one may wonder how the mitigation corresponds to the intensity of signaling. Intuitively, the more intense signaling is, the more downward distortion is mitigated. Unfortunately, with general functional forms, such comparative statics is very complex; indeed, it is even hard to define the intensity of signaling. For tractability, we consider the numerical example below.

Example. Assume that $Q(z,\theta) = \gamma\theta z + z$ with $\gamma > 0$, $C(z,\theta) = z^2 + z - \theta z$, and $\theta \sim U[0,1]$. Using previous results, we have $z^{fb}(\theta) = \frac{(\gamma+1)\theta}{2}$, $z^s(\theta) = \frac{(2\gamma+1)\theta}{2}$ and $z^o(\theta) = \frac{(\gamma+2)\theta-1}{2}$. Define the intensity of signaling to be the ratio of the over-invested education in Spence's model, i.e., $z^s(\theta) - z^{fb}(\theta)$, to the first best education level $z^{fb}(\theta)$ for $\theta > 0$. Substituting, we have

$$\frac{z^s(\theta) - z^{fb}(\theta)}{z^{fb}(\theta)} = \frac{\gamma}{\gamma+1}.$$

Clearly, the intensity of signaling is increasing in the parameter γ. To see the idea, note that the larger γ, the stronger complementarity between the worker's ability and education. In Spence's model, higher education induces the labor market to regard the worker as having higher ability; thus, if ability complements education to a larger extent, the marginal benefit of education will be higher, thereby enhancing signaling through education.

Then, we examine how the signaling intensity affects signaling mitigating the screening distortion. Similarly, we define the extent of the downward distortion in the observed case as the ratio of the under-supplied education, i.e., $z^{fb}(\theta) - z^o(\theta)$, to the first best education level $z^{fb}(\theta)$ for $\theta > 0$. Substituting, we have

$$\frac{z^{fb}(\theta) - z^o(\theta)}{z^{fb}(\theta)} = \frac{1-\theta}{(\gamma+1)\theta}.$$

For any fixed $\theta \in (0,1)$, the extent of the downward distortion is decreasing in γ. This means that the more intense signaling is, the more screening distortion is mitigated.

Recall that in Spence's signaling game, signaling reduces social welfare, as it leads to over-education. In the observed case, by contrast, signaling raises social welfare because it mitigates the screening distortion. From the above analysis, one can infer that if signaling is sufficiently intense, then the welfare loss in Spence's model will exceed that in the observed case; thus, the observed case will yield higher social welfare. To be concrete, we formulate the difference in social welfare between Spence's model and the observed case:

$$\int_{\underline{\theta}}^{\bar{\theta}} \left[S(z^o(\theta),\theta) - S(z^s(\theta),\theta)\right] dF(\theta) = \frac{(\gamma^2+\gamma-1)(\gamma^2+3\gamma+1)}{12(\gamma+2)^2}.$$

It is clear that the observed case yields higher social welfare if and only if $\gamma > \frac{\sqrt{5}-1}{2}$.

This finding has welfare implications for the market structure of signals (which refer to education here). Note that when the market is served by perfectly competitive sellers of signals, the equilibrium outcome is predicted by Spence's model; when the market is served by a monopoly with a publicly observed price schedule, the equilibrium outcome is predicted by the observed case. Therefore, when the buyer's signaling incentive is sufficiently strong, a monopoly can yield higher social welfare than a perfectly competitive market. This implies that introducing competition among signal sellers is not necessarily socially beneficial.

Furthermore, since signaling exerts the opposite welfare effects between Spence's model and the observed case, an instrument that affects the intensity of signaling will also exert the opposite welfare effects between the two cases. Specifically, any instrument that attenuates signaling is socially beneficial in the Spencian world but harmful in the observed case. For example, students' grades substitute for their education levels in signaling; thus, grading is beneficial in the Spencian world but harmful in the observed case. If grades become less informative, e.g., due to grade inflation, then signaling through education will be enhanced, as students will attempt to separate themselves from others (Daley and Green 2014). This reveals that grade inflation is socially beneficial in the observed case by alleviating under-education, while it is harmful in the Spencian world because it aggravates over-education.

5 Labor Market Does Not Observe Tuition

In this section, we turn to the case in which the labor market does not observe the tuition scheme. Given some wage schedule $W(z)$, the school solves the problem in (2.2). Similar to the observed case, we define the school's marginal profit in the unobserved case as

$$MP^u(z,\theta) \equiv W(z) - C(z,\theta) + \frac{1-F(\theta)}{f(\theta)}C_\theta(z,\theta).$$

It is heuristic to call the last two terms the school's *virtual cost*, and we define

$$G(z,\theta) \equiv C(z,\theta) - \frac{1-F(\theta)}{f(\theta)}C_\theta(z,\theta).$$

In doing so, we establish an auxiliary game analogous to Spence's signaling game, in which the worker's cost function is given by $G(z,\theta)$ and utility function by $MP^u(z,\theta)$.

This analogy simplifies the equilibrium characterization of the unobserved case. If there exists an equilibrium with non-decreasing education levels for the auxiliary game, then one can construct an equilibrium for the unobserved case based on that. Specifically, assign the auxiliary game's equilibrium outcome to $\{z^u(\theta), W^u(z)\}$. We conclude that $z^u(\theta)$ solves the school's problem given $W^u(z)$, as it maximizes $MP^u(z,\theta)$ pointwise and is non-decreasing. Moreover, $W^u(z)$ is derived from the correct market belief over $z^u(\theta)$. Thus, $z^u(\theta)$ and $W^u(z)$ satisfy the equilibrium conditions of the unobserved case. As $z^u(\theta)$ has also determined the cutoff type θ_0^u, the tuition scheme $T^u(z)$ can be derived analogously to the observed case.[10] This closes the equilibrium characterization of the unobserved case.

In the following, we instead study the auxiliary game and focus on the school-optimal separating equilibrium. Given Assumption 2.1, we have $G_{z\theta}(z,\theta) < 0$ if $z > 0$, and thus, the single-crossing property holds. This condition means that it is less costly for the school to serve a higher-ability worker. The next proposition shows that the school-optimal separating equilibrium exists in the unobserved case.

[10]Unlike the observed case, it entails some loss of generality to assume that tuition is exorbitantly high for the off-path education, as the school cannot influence the market's belief over the tuition scheme. However, it is natural to smoothly extend $T^u(z)$ to $\mathbb{R}_+$; we show in the Appendix that dosing so is incentive compatible.

Proposition 5.1. *Given Assumption 2.1, the school-optimal separating equilibrium exists, such that*

(i) $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$*;* $z^u(\theta)$ *satisfies the first-order condition*

$$Q_z(z^u(\theta), \theta) + Q_\theta(z^u(\theta), \theta) \cdot \theta^{u\prime}(z^u(\theta)) - G_z(z^u(\theta), \theta) = 0, \tag{5.1}$$

where $\theta^u(z)$ *is the inverse function of* $z^u(\theta)$*, being differentiable on* $[\theta_0^u, \bar{\theta}]$*.*

(ii) $z^u(\theta)$ *is increasing over* $[\theta_0^u, \bar{\theta}]$*, and thus,* $W^u(z^u(\theta)) = Q(z^u(\theta), \theta)$ *for all* $\theta \in [\theta_0^u, \bar{\theta}]$*.*

Proposition 5.1 characterizes the equilibrium education function $z^u(\theta)$. It indicates that the cutoff type and his education level coincide for both the observed and unobserved case. In Appendix A.1, we show that if there is no exclusion in the observed case (i.e., $\theta_0^o = \underline{\theta}$), then the unobserved case has a unique separating equilibrium outcome, which is given above; otherwise (i.e., $\theta_0^o > \underline{\theta}$) there exists a continuum of separating equilibrium outcomes, and in each of them, we have $\theta_0^u \geq \theta_0^o$ and $z^u(\theta_0^u) \geq z^o(\theta_0^o)$. In addition, it is shown in Appendix A.2 that the school-optimal separating equilibrium is also the least-cost separating equilibrium in the sense that the cutoff type chooses his full-information optimal education level under the total cost function $T^u(z) + C(z, \theta)$.

The next theorem presents the paper's main result. In contrast with the observed case, the worker chooses more education in the unobserved case. In particular, a worker who has a higher ability than the cutoff type chooses strictly more education in the unobserved case.

Theorem 5.1. *In contrast with the observed case, the worker chooses more education in the unobserved case. Specifically,* $z^u(\theta) \geq z^o(\theta)$ *on* $[\underline{\theta}, \bar{\theta}]$*, with strict inequality for* $\theta > \theta_0^u$*.*

Proof. Given Assumption 2.1, $MP^o(z, \theta)$ is strictly quasiconcave in z. Because $z^o(\theta)$ is the unique maximizer of $MP^o(z, \theta)$, it suffices to prove that $MP_z^o(z^u(\theta), \theta) \leq 0$, with strict inequality for $\theta > \theta_0^u$. This is given by the following:

$$\begin{aligned} MP_z^o(z^u(\theta), \theta) &= S_z(z^u(\theta), \theta) + \frac{1 - F(\theta)}{f(\theta)} C_{z\theta}(z^u(\theta), \theta) \\ &= Q_z(z^u(\theta), \theta) - G_z(z^u(\theta), \theta) \\ &\leq Q_z(z^u(\theta), \theta) + Q_\theta(z^u(\theta), \theta) \cdot \theta^{u\prime}(z^u(\theta)) - G_z(z^u(\theta), \theta) \\ &= 0. \end{aligned}$$

The second equality is given by the definition of $G(z, \theta)$; the inequality results from the monotonicity of $z^u(\theta)$ on $[\theta_0^u, \bar{\theta}]$; the last equality is due to (5.1). Furthermore, for $\theta > \theta_0^u$, the second term in (5.1) is positive, and thus, the above inequality becomes strict. □

As we informally argued in Section 2 that the school has a lower equilibrium payoff in the unobserved case, this argument is formally proven by the corollary below.

Corollary 5.1. *In the unobserved case, the school's expected profit* Π^u *is strictly lower than its expected profit* Π^o *in the observed case.*

Proof. From Proposition 5.1, we have $MP^u(z^u(\theta), \theta) = MP^o(z^u(\theta), \theta)$. Since $z^o(\theta)$ is the unique maximizer of $MP^o(z, \theta)$ and $z^u(\theta) > z^o(\theta)$ for $\theta > \theta_0^u = \theta_0^o$, we have

$$\Pi^o - \Pi^u = \int_{\theta_0^o}^{\bar{\theta}} [MP^o(z^o(\theta), \theta) - MP^o(z^u(\theta), \theta)] dF(\theta) > 0.$$

Thus, the school is worse off in the unobserved case. □

The difference between the observed and unobserved case is driven by a signal jamming effect. The worker's signal is "jammed" in the unobserved case since the labor market does not observe the actual cost of education. Specifically, the labor market cannot distinguish the impact of a change in tuition from that of cost heterogeneity on the change in education. To illustrate, suppose that the school lowers tuition so that the worker chooses more education than in the initial state. When the labor market observes the tuition change, it cuts wages, as any education level now corresponds to a lower-ability worker. In contrast, when the labor market does not observe the tuition change, it does not adjust wages despite that tuition changes; thus, the worker is willing to pay more for additional education. Conversely, if the school raises tuition such that education decreases, then the labor market will *raise* wages in the observed case; thus, the worker's willingness to pay is lower in the unobserved case. This reveals that the worker is more sensitive to tuition changes in the unobserved case.

From the school's perspective, the demand is more elastic in the unobserved case. Note that the LHS of (5.1) represents the marginal profit of education in the unobserved case; the second term represents the signal jamming effect and is positive. In comparison, in the observed case, rearranging the first-order condition of $MP^o(z, \theta)$, we have

$$MP_z^o(z^o(\theta), \theta) = Q_z(z^o(\theta), \theta) - G_z(z^o(\theta), \theta).$$

Thus, the school's marginal profit is higher in the unobserved case than in the observed case. This provides the school with an incentive to "fool" the labor market with secret price cuts; that is, the school secretly supplies more education and persuades the labor market that the worker is more productive than is actually the case. In equilibrium, the labor market correctly anticipates the school's incentive and offers lower wages, as education is inflated. This reduces the worker's willingness to pay, and thus, the school achieves lower profits.

5.1 Implications for Tuition Transparency

We have shown that tuition cuts lead to smaller increases in demand in the observed case than in the unobserved case. This is because when the tuition cuts are publicly observed, the increase in demand is mitigated by the cheaper tuition reducing the signaling value of education. Hence, tuition cuts are less profitable in the observed case. Here, we show further that tuition is always more expensive in the observed case. Specifically, the tuition scheme in the unobserved case is uniformly lower than that in the observed case over the common domain of education. This is illustrated in Panel (a) of Figure 3.

Proposition 5.2. *$T^u(z) \le T^o(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$, with strict inequality for $z > z^o(\theta_0^o)$.*

Proof. From the worker's first-order condition in both cases, we have

$$\frac{d}{dz}[W^o(z) - T^o(z)] = C_z(z, \theta^o(z)) \quad \text{and} \quad \frac{d}{dz}[W^u(z) - T^u(z)] = C_z(z, \theta^u(z)).$$

According to Theorem 5.1, $z^o(\theta) \le z^u(\theta)$ on $[\theta_0^o, \bar{\theta}]$. As both $z^o(\theta)$ and $z^u(\theta)$ are increasing, $\theta^o(z) \ge \theta^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. Hence, $C_z(z, \theta^o(z)) \le C_z(z, \theta^u(z))$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. This implies that $W^o(z) - T^o(z) \le W^u(z) - T^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. Since $W^o(z) = Q(z, \theta^o(z))$ and $W^u(z) = Q(z, \theta^u(z))$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$, $W^o(z) \ge W^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. Hence, it is readily confirmed that $T^o(z) \ge T^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. □

Furthermore, from the worker's first-order condition in the unobserved case, we have

$$T^{u\prime}(z) = W^{u\prime}(z) - C_z(z, \theta^u(z)).$$

Substituting this equation into (5.1), and noticing that $W^u(z) = Q(z, \theta^u(z))$, we obtain

$$T^{u\prime}(z) = \frac{1 - F(\theta^u(z))}{f(\theta^u(z))} \left[-C_{z\theta}(z, \theta^u(z))\right]. \tag{5.2}$$

Equation (5.2) states that in the unobserved case, the marginal tuition equals the marginal information rent extracted by the worker. In contrast to the observed case, as indicated by the comparison between (5.2) and (4.3), the optimal tuition scheme in the unobserved case does not undo the signaling effect. The reason is that the loss in the social surplus caused by over-education will be compensated by the labor market overpaying the worker, as the labor market will overestimate the worker's ability if the school secretly cuts tuition. In addition, (5.2) states that the marginal tuition vanishes at the highest education level. This implies that the school offers quantity discounts (i.e., $T(z)/z$ is declining) for higher education levels in the unobserved case. This echoes the classic screening model of Maskin and Riley (1984), in which quantity discounts are also optimal at the right tail of the distribution.

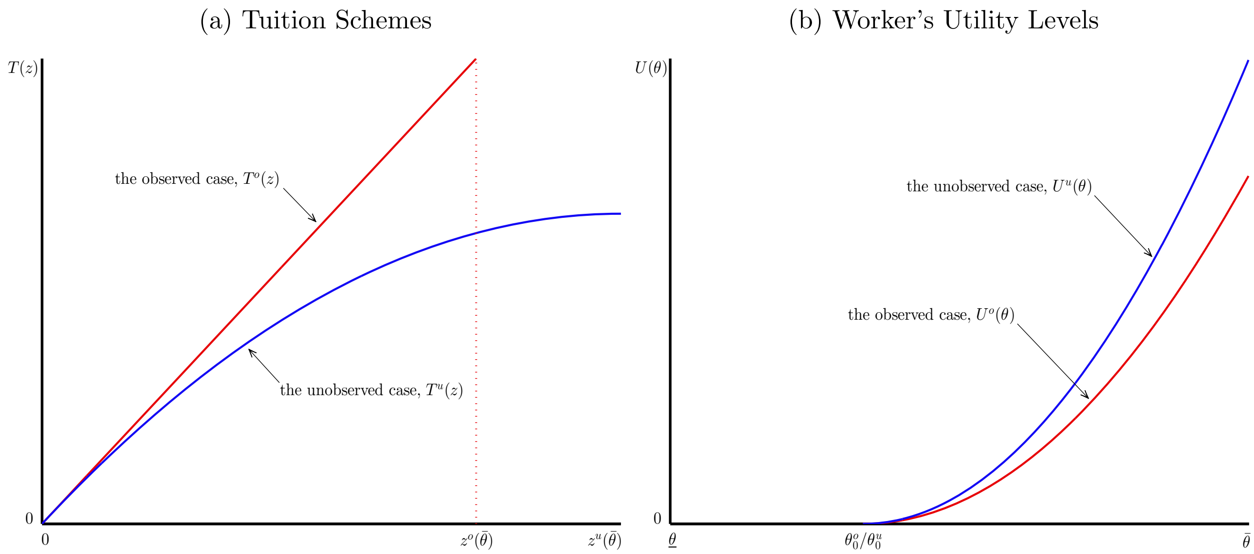

Figure 3: **Implications for Tuition Transparency.** This figure compares tuition rates and the worker's utility level between the observed and unobserved case. This figure considers the same numerical example as Figure 1, such that *(a)* $T^o(z) = \frac{z}{3}$ and $T^u(z) = -\frac{z^2}{4} + \frac{z}{3}$; *(b)* $U^o(\theta) = \frac{3}{4}(\theta - \frac{1}{3})^2$ and $U^u(\theta) = (\theta - \frac{1}{3})^2$.

In terms of the worker's payoff, note that in both cases, the market belief about tuition is correct in equilibrium; thus, given the equilibrium tuition scheme, the continuation game is indeed Spence's signaling game as if the worker's cost function was given by the total cost. Because the tuition scheme is uniformly lower in the unobserved case, the signaling costs are lower in this case. Consequently, the worker has a higher utility level in the unobserved case than in the observed case. This is illustrated in Panel (b) of Figure 3. Formally, we have

Proposition 5.3. *$U^u(\theta) \geq U^o(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality for $\theta > \theta_0^o$.*

Proof. For $\theta \in (\theta_0^o, \bar{\theta}]$, by Lemma 2.1 and Theorem 5.1, we have

$$U^u(\theta) - U^o(\theta) = \int_{\theta_0^o}^{\theta} [C_\theta(z^o(s), s) - C_\theta(z^u(s), s)] ds > 0.$$

The inequality is due to $C_{z\theta} < 0$ and $z^o(\theta) < z^u(\theta)$. For $\theta \in [\underline{\theta}, \theta_0^o]$, $U^u(\theta) = U^o(\theta) = 0$. □

Propositions 5.2 and 5.3 imply that policies that improve the transparency of net prices at colleges and universities through mandatory disclosure may unintentionally induce more expensive education and harm students. These policies, such as U.S. Code § 1015a, require schools to publicly disclose their net prices, which are often not previously observed by employers. Such an intervention allows schools to commit to high prices and not dilute the signaling value of a high-cost education by means of fee waivers, financial aid, and so forth. Hence, the net prices that students actually pay may be higher under such policies.

5.2 Welfare and Education Comparison

We now conduct a welfare analysis for the unobserved case. As a reference point, note that $z^o(\theta_0^o) < z^{fb}(\theta_0^o)$ and $z^o(\bar{\theta}) = z^{fb}(\bar{\theta})$. Because $z^u(\theta) \geq z^o(\theta)$, holding strictly for $\theta > \theta_0^o$, continuity implies that $z^u(\theta)$ intersects $z^{fb}(\theta)$ from below at least once. Moreover, under some mild conditions—the following Assumption 5.1, for example—we show that $z^u(\theta)$ is single-crossing $z^{fb}(\theta)$, i.e., there is a unique cutoff type such that all lower types obtain less education than the first-best while the others obtain more than the first-best (see Figure 4).

Assumption 5.1. *The function*

$$Q_\theta(z^{fb}(\theta), \theta) \cdot \theta^{fb\prime}(z) + \frac{1 - F(\theta)}{f(\theta)} C_{z\theta}(z^{fb}(\theta), \theta) \qquad (*)$$

is single-crossing in θ.[11]

Proposition 5.4. *Given Assumption 5.1, there exists a unique type $\theta^w \in (\theta^*, \bar{\theta})$ such that $z^u(\theta) < z^{fb}(\theta)$ on $[\underline{\theta}, \theta^w)$ and $z^u(\theta) > z^{fb}(\theta)$ on $(\theta^w, \bar{\theta}]$, where $\theta^* > \theta_0^u$ is the root of $(*)$.*

In the unobserved case, there are two competing forces that pull the education function away from the first-best benchmark. On the one hand, the signal jamming effect provides the school with an incentive to supply more education. On the other hand, more education means more information rents to the worker. Since the cost of information rents ultimately vanishes as type approaches the top, the school unambiguously over-supplies education on some upper interval of the spectrum. Assumption 5.1 ensures that the relative significance of the two forces alters only once, thus it rules out the possibility of multiple intersections between $z^u(\theta)$ and $z^{fb}(\theta)$. Proposition 5.4 reveals that under-education is slighter on a lower interval of the spectrum in the unobserved case than in the observed case; it also provides a lower bound for the length of this interval.

However, since over-education also occurs in the unobserved case, whether the observed or unobserved case yields higher social welfare remains ambiguous. Heuristically, if there is slight under-education in the observed case, then over-education will be a relatively more serious issue in the unobserved case; thus, the observed case will yield higher social welfare. Recall that in the observed case, the more intense signaling is, the slighter under-education there is. Thus, the more intense signaling is, the greater over-education in the unobserved case, as the school will find it more profitable to fool the labor market by secretly suppling more education. To illustrate, we revisit the example in Section 4.

[11] A function $g(x)$ is single-crossing in x if given some x^*, $g(x) < 0$ for $x < x^*$ and $g(x) > 0$ for $x > x^*$.

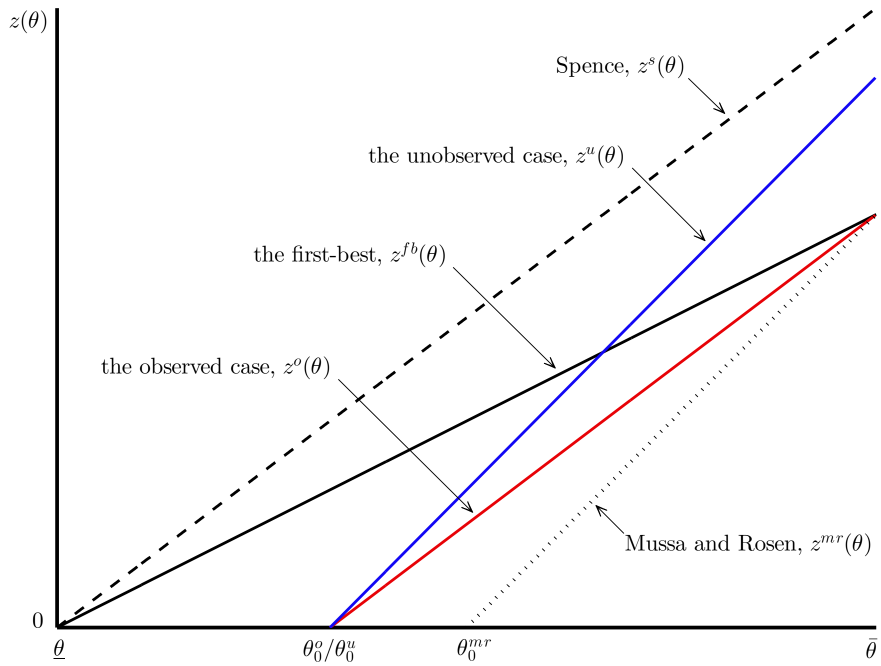

Figure 4: **All Education Functions.** This figure illustrates all the equilibrium education functions that we have discussed in this paper. This figure considers the same numerical example as Figure 1, such that $z^u(\theta) = 2\theta - \frac{2}{3}$, and recall that $z^{fb}(\theta) = \theta$, $z^s(\theta) = \frac{3}{2}\theta$, $z^o(\theta) = \frac{3\theta-1}{2}$ and $z^{mr}(\theta) = 2\theta - 1$.

Example. Assume that $Q(z,\theta) = \gamma\theta z + z$ with $\gamma > 0$, $C(z,\theta) = z^2 + z - \theta z$, and $\theta \sim U[0,1]$. Applying Proposition 5.1, we have $z^u(\theta) = (\gamma+1)\theta - \frac{\gamma+1}{\gamma+2}$. Recall that $z^{fb}(\theta) = \frac{(\gamma+1)\theta}{2}$ and $z^o(\theta) = \frac{(\gamma+2)\theta-1}{2}$. It is readily confirmed that the welfare cutoff type $\theta^w = \frac{2}{\gamma+2}$, which is decreasing in γ. This implies that the over-education region is increasing in the intensity of signaling. Moreover, the difference in social welfare between the two cases is given by

$$\int_{\theta_0^o}^{\bar{\theta}} [S(z^o(\theta),\theta) - S(z^u(\theta),\theta)]\, dF(\theta) = \frac{\gamma(\gamma-1)(\gamma+1)^3}{12(\gamma+2)^3}.$$

Clearly, the RHS is positive if and only if $\gamma > 1$; that is, if signaling is sufficiently intense, then the observed case yields higher social welfare than the unobserved case.

Our last proposition indicates that the education function in the unobserved case $z^u(\theta)$ is bounded above by that of Spence's signaling game $z^s(\theta)$. This is illustrated in Figure 4.

Proposition 5.5. *In the unobserved case, the worker chooses strictly less education than in Spence's signaling game, that is, $z^u(\theta) < z^s(\theta)$ on $[\underline{\theta}, \bar{\theta}]$.*

The intuition is straightforward, as the unobserved case is essentially Spence's signaling game with higher costs, meaning that it yields lower education levels. Proposition 5.5 implies that if signaling is sufficiently intense so that over-education is prevalent in the unobserved case, then social welfare is higher in the unobserved case than in Spence's model, as the former case yields slighter over-education than the latter.[12]

In this paper, we performed a pairwise comparison of different education functions. We first showed that signaling alone leads to over-education, i.e., $z^s(\theta) > z^{fb}(\theta)$. Then, after accounting for the school's strategic behavior, the equilibrium education functions vary with the labor market's information. The table below summarizes the correspondence between the equilibrium education function and information structure.

Table 1: **Education Functions under Different Information Structures.**

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">Labor Market Observes Tuition</td></tr>
<tr><td>No</td><td>Yes</td></tr>
<tr><td rowspan="2">Labor Market Observes Type</td><td>No</td><td>$z^u(\theta)$</td><td>$z^o(\theta)$</td></tr>
<tr><td>Yes</td><td colspan="2">$z^{mr}(\theta)$</td></tr>
</table>

As illustrated by Table 1, when the labor market observes the worker's ability and the school's tuition scheme, the model is Mussa and Rosen's screening game. A higher-ability worker benefits from his productivity and cost advantage over others. To incentivize truth-telling, the school leaves information rents to the worker and thus under-supplies education, that is, $z^{mr}(\theta) < z^{fb}(\theta)$. When the labor market observes only the tuition scheme, a higher-ability worker cannot benefit directly from his productivity advantage, and thus, signaling arises. Signaling mitigates the screening distortion since the school incurs lower information rents that stem from worker cost heterogeneity only. Thus, $z^{mr}(\theta) < z^o(\theta) < z^{fb}(\theta)$. Finally, when the labor market observes neither the tuition scheme nor the worker's ability, the worker becomes more sensitive to tuition changes, and thus, the demand for education is more elastic than in the observed case. This makes price cuts relatively more profitable for the school and induces it to supply more education; therefore, $z^o(\theta) < z^u(\theta)$.

[12]In the previous numerical example, if and only if γ is larger than some cutoff that is less than $\frac{\sqrt{5}-1}{2}$, the unobserved case yields higher social welfare than Spence's model. Thus, we have derived all the three cutoffs for a pairwise welfare comparison between Spence's model, the observed and unobserved case. These cutoffs partition the domain of γ into four divisions in which the three cases rank differently in terms of social welfare. It is clear that as the intensity of signaling raises (i.e., γ increases), the case that yields the highest social welfare will be, respectively, Spence's model, the unobserved case and the observed case.

Remark. If education is a pure signal, all the results in this section are valid. Specifically, in the school-optimal separating equilibrium, the lowest type has no education and the others obtain increasing and positive amounts of education. Recall that in the observed case, the lowest type has zero education and the other types obtain infinitesimal education. Thus, our main result still holds. Moreover, in the unobserved case the school gains lower profits while the worker has a higher utility level than in the observed case. Finally, because education is unproductive and the unobserved case yields higher education levels, social welfare is higher in the observed case. See Appendix A.3 for further details.

6 Summary and Discussion

In this paper, we developed classic signaling models by allowing a third party to affect the signaling cost. We used this framework to analyze how a school with market power, e.g., a top business school, manages job market signaling by designing a tuition scheme. The equilibrium depends critically on whether employers observe the tuition scheme. In the observed case, the school internalizes the worker's signaling incentive when screening his type, causing under-education. In the unobserved case, the worker's signal is jammed and he is more sensitive to tuition changes. This leads to a more elastic demand for education and induces the school to lower tuition rates. In equilibrium, the worker chooses more education and obtains higher utility than in the observed case, whereas the school achieves lower profits than in the observed case.

Our framework has policy implications for the transparency of tuition at colleges and universities. Mandatory disclosure policies, such as U.S. Code § 1015a, make the net prices of education public information. On the one hand, this reduces the search costs of students, thereby stimulating the competition between schools and lowering prices; on the other hand, this also allows schools to commit to high prices and not dilute the signaling value of a high-cost education by means of fee waivers, financial aid and so forth. It is thus possible that such policies ultimately raise education costs and harm students. Hence, policymakers should not overlook the potential drawbacks of these mandatory disclosure policies.

Our framework has welfare implications for the market structure of signals. When the market is served by perfectly competitive sellers, the equilibrium outcome is predicted by Spence's model; when the market is served by a monopoly with a publicly observed price schedule, the equilibrium outcome is predicted by the observed case. We show that when the buyer's signaling incentive is sufficiently strong, a monopoly can yield higher social welfare than a perfectly competitive market. This implies that introducing competition among the sellers of signals is not necessarily socially beneficial.

Our framework also draws attention to the positive side of grade inflation at colleges and universities. Grade inflation, which is documented by a large body of empirical work (e.g., Johnson 2006, Rojstaczer and Healy 2010), induces students to obtain more education to signal their intrinsic and unobserved abilities. Our model suggests that when schools have market power and under-supply education due to screening, grade inflation can mitigate the screening distortion by encouraging education, thereby raising social welfare.

6.1 Applications of the Model

In addition to job market signaling, our model can be applied to other vertical relationships in which signaling prevails, such as conspicuous consumption and advertising. In the case of conspicuous consumption, a retailer (*principal*) chooses a price schedule $T(z)$ for a luxury good, where z denotes the quality of the good. Then, as in Bagwell and Bernheim (1996), a consumer (*agent*) chooses the quality of the good he will purchase to signal his unobserved wealth (*type*) θ to the social contact (*market*); the social contact observes z and forms some belief about θ. In the spirit of the seminal work of Veblen (1899), the social contact rewards the consumer based on z. The reward scheme $W(z)$ is given by the social contact's expected benefit $\mathbb{E}[Q(z,\theta)]$ from the consumer; the function $Q(z,\theta)$ is increasing in both arguments, as the social contact obtains higher utility by making friends with richer people and sharing goods of higher quality with them. Moreover, the consumer derives intrinsic utility from the luxury good. The intrinsic utility is denoted by $V(z,\theta)$, which is increasing in the quality z. More important, the single-crossing condition holds: $V_{z\theta}(z,\theta) > 0$. This condition captures the feature that a wealthier individual has higher marginal utility from consuming a luxury good. For example, a buyer of a yacht can voyage more often if he is richer, as he is better able to afford the fuel costs and maintenance fees. In terms of payoffs, the retailer's profit equals the revenue $T(z)$ minus the cost $C(z)$; the consumer's net utility equals the reward $W(z)$ plus the intrinsic utility $V(z,\theta)$ and minus the price $T(z)$. Then, given such a similar setup, one can easily replicate the analysis we have conducted in the education application.

We now turn to the application of advertising. In this case, a media company (*principal*) chooses a price schedule $T(z)$ for advertising messages, where z denotes advertising level. Then, as in Milgrom and Roberts (1986), a producer (*agent*) that has just developed a new product chooses its advertising level to signal the unobserved quality (*type*) θ of the product to consumers (*market*); consumers observe z and form some belief about θ. The producer's revenue has two sources: the purchase in the introductory stage and the repeat purchase in the post-introductory stage. The introductory revenue $R^i(z, \hat{\theta})$ depends on the advertising level z and the expected quality $\hat{\theta}$, and is increasing in both arguments, as more advertising results in higher consumer awareness, and better consumer perception allows the producer to charge a higher price. In contrast, the post-introductory revenue $R^p(z, \theta)$ depends on the actual quality θ instead of $\hat{\theta}$, since the product's quality is revealed after the introductory stage. $R^p(z, \theta)$ is increasing in z, as more introductory advertising results in a larger base of repeat purchase. More important, the single-crossing condition holds: $R^p_{z\theta}(z, \theta) > 0$. This is due to the complementarity between advertising and quality; that is, the marginal revenue of the introductory advertising is higher if the product is of higher quality, thereby allowing the producer to charge higher prices in the post-introductory stage. In terms of payoffs, the media company's profit equals the revenue $T(z)$ minus the cost $C(z)$; the producer's profit equals the sum of the introductory revenue $R^i(z, \hat{\theta})$ and post-introductory revenue $R^p(z, \theta)$ minus the price $T(z)$, with production costs normalized to zero. In particular, if $R^i(z, \hat{\theta})$ is linear in $\hat{\theta}$, then $R^i(z, \hat{\theta}) = \mathbb{E}[R^i(z, \theta)]$, and thus, the setup is similar to the education application and the analysis will be analogous.

A remark on the advertising model is as follows: the introductory price that is chosen by the producer may also be used as a signal of quality, as in Milgrom and Roberts (1986). In this regard, the producer faces a trade-off between signaling by price and signaling by advertising, depending on which channel is more effective; it is possible that both types of signal coexist in equilibrium. In turn, the possibility of multiple signals will also affect the media company's pricing strategy, making the analysis more complicated.

6.2 Extensions of the Model

To isolate the impacts of pricing transparency on the degree of signaling and welfare, we assumed in this paper that the principal is a monopolist and the agent has a zero-utility outside option. In the project we are working on, we further investigate how (horizontal) competition affects the principal's pricing strategy and the degree of signaling for both the observed and unobserved case. In contrast with the current model, the agent's preference is two-dimensional: the vertical preference parameter is in conformity with the current model; the horizontal preference parameter captures the agent's outside option; these parameters are independent and both privately known. In this case, the agent has a positive and type-dependent outside option; thus, the principal has an incentive to charge lower prices than in the current model to gain the market share of each vertical type.

We show that the extent of the price decrease is relatively greater in the observed case than in the unobserved case. Specifically, the price schedule in the observed case is lower than that in the unobserved case at the left tail of the common domain; thus, an interval of vertical types at the low end of the spectrum choose higher quantities and obtain higher utilities in the observed case. The reason is that in the unobserved case the signal jamming effect induces the principal to cut prices, as in the current paper; if the principal charges as low of prices as in the observed case for those lower quantities, then it must further lower the prices for higher quantities to remain incentive compatibility, which is unprofitable. We show that the length of such an interval is increasing in the agent's outside option, in terms of an exogenous parameter. In particular, when the parameter reaches zero, we restore all the results of the current model. This means that there is no discontinuity in the current paper's results when we disturb the participation constraint somewhat. We also analyze an oligopoly case in which multiple principles are competing in selling signals. We show that when all vertical types are served in equilibrium, each principal offers a *cost-plus-fixed-fee* tariff, thereby eliminating screening whereas preserving signaling. The equilibrium result is thus analogous to that of Spence's model.

The current paper's results still hold if we change non-linear tuition to linear tuition or change continuous types to discrete types. A somewhat special case is linear tuition with discrete types. Without loss of generality, suppose that there are only two types, low and high, and the school chooses a uniform tuition rate. In the observed case, the least-cost separating equilibrium exists, in which the high type obtains more education than the low type, and the latter is indifferent between revealing own type and imitating the former. In the unobserved case, however, such an equilibrium does not exist because the high education level is so high that the low type strictly prefers to reveal his type. The intuition is similar: the high education level must be relatively too high for the low type to imitate the high type, such that the school finds it unprofitable to cut the price and gain market share.

A new economic force arises in this case due to the school's inability to price discriminate. That is, when the high type strictly prefers to separate himself from the low type, the school has an incentive to squeeze him by raising tuition. The reason is that the high type is less sensitive to tuition changes, as a decrease in education will cause him to be regarded as the low type even if this decrease is due to higher tuition. Therefore, the school faces a trade-off between squeezing the high type and maintaining the low type's market share. If the gap between the two types and the proportion of the high type are large enough, squeezing the high type is relatively more profitable, such that in equilibrium the low type is excluded from education and the high type is indifferent between choosing the equilibrium high education level and deviating downward optimally.

A Appendix

A.1 Omitted Proofs

Proof of Proposition 3.1.

Proof. Let $U(\theta, \hat{\theta}, z)$ be type-θ's payoff if he chooses education level z and is believed as type-$\hat{\theta}$. In particular, $U(\theta, \theta, z)$ equals $S(z, \theta)$, which is strictly quasiconcave in z and has a unique maximizer. Moreover, $U_2(\theta, \hat{\theta}, z) = Q_\theta(z, \hat{\theta}) > 0$, $U_{13}(\theta, \hat{\theta}, z) = -C_{z\theta}(z, \theta) > 0$, and $U_3(\theta, \hat{\theta}, z)/U_2(\theta, \hat{\theta}, z)$ is increasing in θ. Appealing to Mailath (1987, Theorem 3), we prove that a function $z(\theta)$ is incentive compatible if it satisfies (i) and (ii) of Proposition 3.1 and such a function uniquely exists given the initial condition. We assume that the market holds the worst belief off the equilibrium path, so that no type is willing to deviate to there. Thus, the least-cost separating equilibrium exists and is characterized by Proposition 3.1. □

Proof of Proposition 5.1.

Proof. We first prove that a separating equilibrium exists in the unobserved case. Fix some admissible initial point $(\theta_0^u, z^u(\theta_0^u))$. Given Assumption 2.1, $MP^o(z, \theta)$ is regular. Replacing $C(z, \theta)$, $U(z, \theta)$ and $S(z, \theta)$ by $G(z, \theta)$, $MP^u(z, \theta)$ and $MP^o(z, \theta)$, respectively, we obtain immediately the existence by replicating the proof of Proposition 3.1.

To find the school-optimal separating equilibrium, it suffices to pin down the initial point. We consider two cases. First, $\theta_0^o = \underline{\theta}$. Note that the lowest possible wage for any education level $z > 0$ is $Q(z, \underline{\theta})$. Thus, for every pair (z, θ) with $z > 0$, we have

$$MP^u(z, \theta) \geq Q(z, \underline{\theta}) - G(z, \theta) \geq Q(z, \underline{\theta}) - G(z, \underline{\theta}) = MP^o(z, \underline{\theta}).$$

The second inequality is due to $G_{z\theta} < 0$ if $z > 0$. Since $\theta_0^o = \underline{\theta}$, $MP^u(z^o(\underline{\theta}), \underline{\theta}) \geq 0$; that is, the marginal profit of the lowest type can be at least non-negative. Thus, $\theta_0^u = \underline{\theta}$. Note too that $MP^u(z^u(\theta), \theta) = MP^o(z^u(\theta), \theta)$, as types reveal in equilibrium. Then, it is optimal for the school to choose $z^u(\underline{\theta}) = z^o(\underline{\theta})$, because the labor market cannot punish this choice by holding a worse belief than $\underline{\theta}$ and $z^o(\underline{\theta})$ maximizes $MP^o(z, \underline{\theta})$ by definition. Thus, if $\theta_0^o = \underline{\theta}$, then the separating equilibrium outcome is unique such that $(\theta_0^u, z^u(\theta_0^u)) = (\underline{\theta}, z^o(\underline{\theta}))$.

Second, $\theta_0^o > \underline{\theta}$. In this case, $(\theta_0^u, z^u(\theta_0^u))$ and thus the equilibrium outcome is not unique. From Mailath (1987, Theorem 3), for every separating equilibrium, $z^u(\theta)$ satisfies (5.1) and is increasing. Analogously to the proof of Theorem 5.1, we have $z^u(\theta) \geq z^o(\theta)$ on $[\theta_0^u, \bar{\theta}]$ with strict inequality for $\theta > \theta_0^u$. This implies that $MP^u(z^u(\theta), \theta) \leq MP^o(z^o(\theta), \theta)$, as $z^o(\theta)$ is the unique maximizer of $MP^o(z, \theta)$, and $MP^u(z^u(\theta), \theta) = MP^o(z^u(\theta), \theta)$. By the definition of the cutoff type, we have $\theta_0^u \geq \theta_0^o$ in every separating equilibrium of the unobserved case. Thus, we have determined the lower bound of $(\theta_0^u, z^u(\theta_0^u))$. In Appendix A.2, we show that the school-optimal separating equilibrium exists in this case such that $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$. In summary, in both cases, $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$; thus, Proposition 5.1 is proven. □

Proof of Proposition 5.4.

Proof. We only need to study the interval $(\theta_0^u, \bar{\theta})$. We have shown that $z^u(\theta)$ intersects $z^{fb}(\theta)$ from below at least once. Note that $z^u(\theta_0^u) = z^o(\theta_0^u) < z^{fb}(\theta_0^u)$ and $z^u(\bar{\theta}) > z^o(\bar{\theta}) = z^{fb}(\bar{\theta})$. If there are multiple intersections, then $z^u(\theta)$ intersects $z^{fb}(\theta)$ at least three times. Denote by $W^{fb}(z)$ the wage schedule in the first-best benchmark. Since both $W^u(z)$ and $W^{fb}(z)$ are increasing, it suffices to prove that $W^u(z)$ intersects $W^{fb}(z)$ only once. Suppose that $z^u(\theta)$ intersects $z^{fb}(\theta)$ at some θ^w, then $W^u(z)$ intersects $W^{fb}(z)$ at $z^{fb}(\theta^w)$. Differentiating both $W^u(z)$ and $W^{fb}(z)$ at $z^{fb}(\theta^w)$, respectively, we have

$$\begin{aligned} W^{u\prime}(z^{fb}(\theta^w)) &= C_z(z^{fb}(\theta^w), \theta^w) - \frac{1 - F(\theta^w)}{f(\theta^w)} C_{z\theta}(z^{fb}(\theta^w), \theta^w), \\ W^{fb\prime}(z^{fb}(\theta^w)) &= Q_z(z^{fb}(\theta^w), \theta^w) + Q_\theta(z^{fb}(\theta^w), \theta^w) \cdot \theta^{fb\prime}(z^{fb}(\theta^w)). \end{aligned}$$

The first equation results from the first-order condition of $MP^u(z, \theta)$; the second is just the total derivative of $W^{fb}(z)$. Rearranging and substituting (2.1), we have

$$W^{fb\prime}(z^{fb}(\theta^w)) - W^{u\prime}(z^{fb}(\theta^w)) = Q_\theta(z^{fb}(\theta^w), \theta^w) \cdot \theta^{fb\prime}(z^{fb}(\theta^w)) + \frac{1 - F(\theta^w)}{f(\theta^w)} C_{z\theta}(z^{fb}(\theta^w), \theta^w).$$

Given Assumption 5.1, the RHS can change its sign only once for different values of θ^w. Suppose that $z^u(\theta)$ intersects $z^{fb}(\theta)$ more than once, then the directions of the first three intersections are from below, from above, and from below; thereby, $W^u(z)$ intersects $W^{fb}(z)$ first from above, then from below, and then from above. This means that the LHS of the above equation will change its sign more than once, a contradiction. Thus, we conclude that $z^u(\theta)$ intersects $z^{fb}(\theta)$ only once and from below. Then, $W^{fb\prime}(z^{fb}(\theta^w)) - W^{u\prime}(z^{fb}(\theta^w)) > 0$, meaning that the RHS of the above equation is positive. By the definition of θ^*, we have

$$Q_\theta(z^{fb}(\theta^*), \theta^*) \cdot \theta^{fb\prime}(z^{fb}(\theta^*)) + \frac{1 - F(\theta)}{f(\theta)} C_{z\theta}(z^{fb}(\theta^*), \theta^*) = 0.$$

It is readily confirmed by Assumption 5.1 that $\theta^w > \theta^*$. Thus, the proposition is proven. □

Proof of Proposition 5.5.

Proof. We only need to prove that $z^u(\theta) < z^s(\theta)$ on $[\theta_0^u, \bar{\theta}]$. From the first-order conditions, we can derive $W^u(z)$ and $W^s(z)$, respectively, by the initial value problems (IVP) below:

$$W^{u\prime}(z) = G_z(z, \theta^u(w, z)) \ \text{ and } \ W^{s\prime}(z) = C_z(z, \theta^s(w, z)),$$

with the initial points $(z^o(\theta_0^o), W^u(z^o(\theta_0^o)))$ and $(z^o(\theta_0^o), W^s(z^o(\theta_0^o)))$ for $W^u(z)$ and $W^s(z)$, respectively. It is easy to see that $G_z(z, w) \geq C_z(z, w)$ in any common domain of (z, w). From Corollary 4.1, we have $z^o(\theta) < z^s(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, and thus, $\theta_0^o > \theta^s(z^o(\theta_0^o))$. This implies that $W^u(z^o(\theta_0^o)) > W^s(z^o(\theta_0^o))$. Then, appealing to Hartman (1964, Corollary 4.2, page 27), we have $W^u(z) > W^s(z)$ in any common domain. This implies that $\theta^u(z) > \theta^s(z)$ in any common domain; therefore, $z^u(\theta) < z^s(\theta)$ on $[\theta_0^u, \bar{\theta}]$. □

A.2 Equilibrium Selection for the Unobserved Case

Here, we discuss equilibrium selection for the unobserved case. We present two lemmata. By Lemma A.1, we characterize the school-optimal separating equilibrium given that $\theta_0^o > \underline{\theta}$; by Lemma A.2, we show that the school-optimal separating equilibrium is also the least-cost separating equilibrium with respect to the total cost of education.

Lemma A.1. *Given that $\theta_0^o > \underline{\theta}$, the school-optimal separating equilibrium exists in the unobserved case, such that $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$.*

Proof. As a first step, we show that the cutoff type's education level $z^u(\theta_0^u)$ is an increasing function of θ_0^u. From the proof of Proposition 5.1, we have $\theta_0^u \geq \theta_0^o > \underline{\theta}$. Thus,

$$MP^u(z^u(\theta_0^u), \theta_0^u) = MP^o(z^u(\theta_0^u), \theta_0^u) = 0.$$

Given Assumption 2.1, $MP^o(z, \theta)$ is regular; $z^o(\theta_0^u)$ is the unique maximizer of $MP^o(z, \theta_0^u)$. From the proof of Proposition 5.1, we have $z^u(\theta_0^u) \geq z^o(\theta_0^u)$ for each separating equilibrium. Then, regularity implies that $z^u(\theta_0^u)$ is the unique solution to the above equation given an admissible θ_0^u, and $z^u(\theta_0^u)$ is increasing. Thus, $z^u(\theta_0^u)$ is an increasing function of θ_0^u.

Second, we show that for any two admissible initial points $(\hat{\theta}_0^u, \hat{z}^u(\theta))$ and $(\tilde{\theta}_0^u, \tilde{z}^u(\theta))$, if $\hat{\theta}_0^u < \tilde{\theta}_0^u$, then $\hat{z}^u(\theta) < \tilde{z}^u(\theta)$ in any common domain. From Mailath (1987, Theorem 3), for every separating equilibrium, $z^u(\theta)$ satisfies (5.1). Rearranging (5.1), we have

$$z^{u\prime}(\theta) = \frac{Q_\theta(z^u(\theta), \theta)}{G_z(z^u(\theta), \theta) - Q_z(z^u(\theta), \theta)}.$$

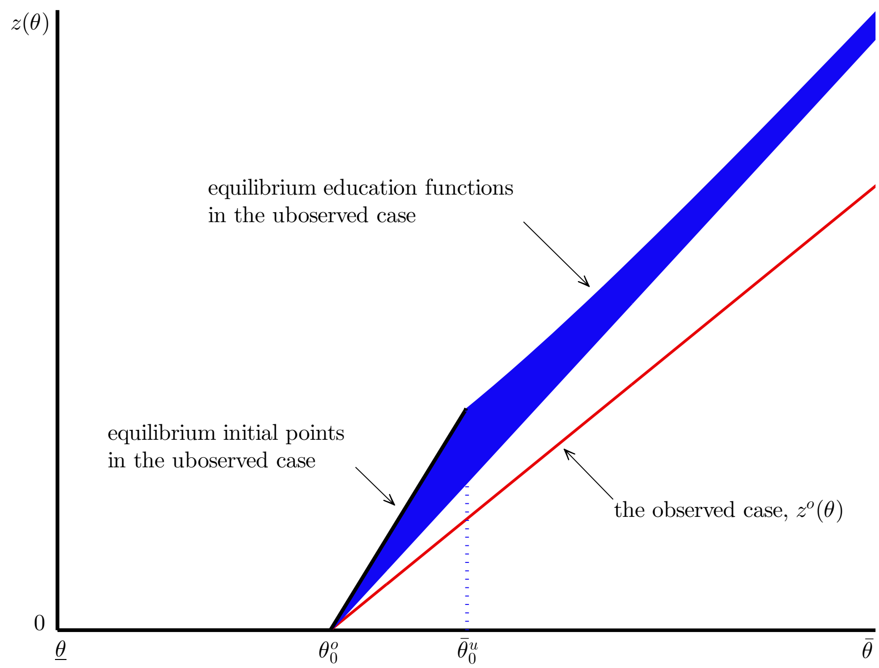

Figure 5: **The Family of Separating Equilibria.** This figure illustrates the family of separating equilibria in the unobserved case given that $\theta_0^o > \underline{\theta}$. The blue area depicts the set of equilibrium education functions. This region is uniformly above the equilibrium education function in the observed case $z^o(\theta)$. The bold line is the set of equilibrium initial points with the cutoff type ranging from θ_0^o to $\bar{\theta}_0^u$. Each point uniquely determines an equilibrium education function $z^u(\theta)$ and thus an equilibrium outcome. This figure considers the same numerical example as Figure 1, such that the set of the initial points is $\{(\theta, z)|z(\theta) = 3\theta - 1; \frac{1}{3} \leq \theta \leq \frac{1}{2}\}$.

From the first paragraph, if $\hat{\theta}_0^u < \tilde{\theta}_0^u$, then $\hat{z}^u(\hat{\theta}_0^u) < \tilde{z}^u(\tilde{\theta}_0^u)$. Appealing to Hartman (1964, Corollary 4.2, page 27), we have $\hat{z}^u(\theta) < \tilde{z}^u(\theta)$ in the common domain $[\tilde{\theta}_0^u, \bar{\theta}]$.

Third, we characterize $z^u(\theta)$ for all separating equilibria. To do so, we have to determine the domain of $z^u(\theta_0^u)$, which depends on the market belief off the equilibrium path. As have been shown, the lower bound of θ_0^u is θ_0^o, which is supportable if any off-path education is believed to be chosen by type θ_0^o. As the off-path belief gets gradually harsher, θ_0^u increases continuously, until the labor market holds the worst belief $\underline{\theta}$ off the equilibrium path. It is without loss of generality to confine the off-path education to $[0, z^u(\theta_0^u))$ when $z^u(\theta_0^u) > 0$. Denote by $\bar{\theta}_0^u$ the upper bound of θ_0^u, which is pinned down by

$$\max_{z < z^u(\bar{\theta}_0^u)} \{Q(z, \underline{\theta}) - G(z, \bar{\theta}_0^u)\} = MP^u(z^u(\bar{\theta}_0^u), \bar{\theta}_0^u) = 0.$$

That is, the school is indifferent between allocating type-$\bar{\theta}_0^u$ the optimal off-path education such that it is believed as type-$\underline{\theta}$ and maintaining the equilibrium allocation. Therefore, we have determined the domain of $z^u(\theta_0^u)$. Then, picking any $\theta_0^u \in [\theta_0^o, \bar{\theta}_0^u]$, we can uniquely pin down a $z^u(\theta)$. Figure 5 illustrates the education functions of all separating equilibria.

Finally, we show that the initial point of the school-optimal separating equilibrium is $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$. Pick two equilibrium education functions, $\hat{z}^u(\theta)$ and $\tilde{z}^u(\theta)$, such that $\hat{z}^u(\theta) < \tilde{z}^u(\theta)$ on the common support $[\tilde{\theta}_0^u, \bar{\theta}]$. Since $z^u(\theta) \geq z^o(\theta)$ on $[\theta_0^u, \bar{\theta}]$ for every separating equilibrium, we have $\hat{z}^u(\theta) - z^o(\theta) < \tilde{z}^u(\theta) - z^o(\theta)$ on $[\tilde{\theta}_0^u, \bar{\theta}]$. Thus, regularity implies that $MP^u(\hat{z}^u(\theta), \theta) > MP^u(\tilde{z}^u(\theta), \theta)$ on $[\hat{\theta}_0^u, \bar{\theta}] \supset [\tilde{\theta}_0^u, \bar{\theta}]$. Then,

$$\Pi^u(\hat{\theta}_0^u) - \Pi^u(\tilde{\theta}_0^u) = \int_{\hat{\theta}_0^u}^{\bar{\theta}} MP^u(\hat{z}^u(\theta), \theta) dF(\theta) - \int_{\tilde{\theta}_0^u}^{\bar{\theta}} MP^u(\tilde{z}^u(\theta), \theta) dF(\theta) > 0.$$

The inequality is due to the fact that both the integrand and the integral domain of $\Pi^u(\hat{\theta}_0^u)$ are bigger than those of $\Pi^u(\tilde{\theta}_0^u)$. This result reveals that the lower the cutoff type, the higher the school's equilibrium payoff. Since $\theta_0^u \in [\theta_0^o, \bar{\theta}_0^u]$, the school-optimal separating equilibrium must be the one in which $\theta_0^u = \theta_0^o$, and thus, $z^u(\theta_0^u) = z^o(\theta_0^o)$. □

Lemma A.2. *In the school-optimal separating equilibrium, the cutoff type θ_0^u chooses his full-information optimal education level under the total cost function $T^u(z) + C(z, \theta_0^u)$, i.e.,*

$$z^u(\theta_0^u) = \underset{z}{argmax} \;\; Q(z, \theta_0^u) - T^u(z) - C(z, \theta_0^u).$$

Proof. First, we characterize $T^u(z)$ on $\mathbb{R}_+$. On the equilibrium path, $T^u(z)$ is given by

$$T^u(z^u(\theta)) = S(z^u(\theta), \theta) - U^u(\theta) = S(z^u(\theta), \theta) + \int_{\theta_0^u}^{\theta} C_\theta(z^u(s), s) ds.$$

Then, we smoothly extend $T^u(z)$ to $\mathbb{R}_+$. First, from (5.2), we have $\lim_{z \to z^u(\bar{\theta})^-} T^{u\prime}(z) = 0$. It is thus natural to extend $T^u(z)$ horizontally upto $+\infty$. Second, if $z^u(\theta_0^u) > 0$, then we smoothly extend $T^u(z)$ to the left by extending the solution to the IVP that is defined by the differential equation in (5.2) and the initial condition that $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$, until $T^u(z)$ or z reaches 0, whichever is earliest. The rest part of $T^u(z)$ is fixed at 0. Thus, $T^u(z)$ is fully characterized on $\mathbb{R}_+$. To ensure that such $T^u(z)$ is incentive compatible, we simply assume that the labor market holds the worst belief $\underline{\theta}$ for any off-path education level, so that no type will deviate to the off-path.

Thus, given $T^u(z)$, it suffices to prove that the following first-order condition holds.

$$Q_z(z^u(\theta_0^u), \theta_0^u) - T^{u\prime}(z^u(\theta_0^u)) - C_z(z^u(\theta_0^u), \theta_0^u) = 0. \quad \text{(A.1)}$$

Note that $MP^o(z, \theta)$ is regular, $z^o(\theta_0^o)$ maximizes $MP^o(z, \theta_0^o)$ and $z^u(\theta_0^u) = z^o(\theta_0^o)$, thus

$$MP_z^o(z^u(\theta_0^u), \theta_0^u) = Q_z(z^u(\theta_0^u), \theta_0^u) - G_z(z^u(\theta_0^u), \theta_0^u) = 0.$$

Substituting $G_z(z^u(\theta_0^u), \theta_0^u)$ using (5.2) and the definition of $G(z, \theta)$, we obtain (A.1). □

A.3 Unproductive Education

Here, we consider the case in which education is a pure signal. Without loss of generality, we assume that $Q(\theta) = \theta > 0$ on $[\underline{\theta}, \bar{\theta}]$. Thus, the social surplus function $S(z, \theta)$ is decreasing in z, meaning that zero education is socially optimal, i.e., $z^{fb}(\theta) \equiv 0$.

We start with the observed case and focus on the school-optimal separating equilibrium. Unlike the productive education case, the worker's outside option is now endogenous, which depends on the off-path belief. Without loss of generality, we assume that the labor market holds the worst belief $\underline{\theta}$ for all positive off-path education levels. The equilibrium wage for zero education, $W(0)$, equals $\mathbb{E}[\theta|\theta \in [\underline{\theta}, \theta_0)]$ with the market belief updated by Bayes' rule. Thus, the worker's outside option is endogenously given by $\mathbb{E}[\theta|\theta \in [\underline{\theta}, \theta_0)]$. Analogously to Section 2.1, the school's problem can be stated as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ S(z(\theta), \theta) + \frac{1 - F(\theta)}{f(\theta)} C_\theta(z(\theta), \theta) - \mathbb{E}[\theta|\theta \in [\underline{\theta}, \theta_0)] \right\} dF(\theta).$$

subject to $z(\theta)$ being increasing on $[\theta_0, \bar{\theta}]$. Substituting $Q(\theta)$ and $G(z, \theta)$ into the integral, we can succinctly write the school's value function as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \{\theta - G(z(\theta), \theta) - \mathbb{E}[\theta|\theta \in [\underline{\theta}, \theta_0)]\} \, dF(\theta).$$

Suppose that in equilibrium $\theta_0 > \underline{\theta}$, then by differentiating the value function with respect to θ_0 and rearranging, we obtain the derivative as

$$f(\theta_0) \cdot G(z(\theta_0), \theta_0) - \frac{f(\theta_0)}{F(\theta_0)} \left(\theta_0 - \mathbb{E}[\theta|\theta \in [\underline{\theta}, \theta_0)]\right).$$

Note that $z(\theta_0)$ must be zero. Suppose not, then $z(\theta) > 0$ for all $\theta \in [\theta_0, \bar{\theta}]$, as $z(\theta)$ is increasing. But since the marginal profit $\theta - G(z, \theta)$ is decreasing in z, it is profitable to reduce all positive $z(\theta)$ by a fixed small amount, a contradiction. Hence, $G(z(\theta_0), \theta_0) = 0$. Note too that $\theta_0 > \mathbb{E}[\theta|\theta \in [\underline{\theta}, \theta_0)]$. Hence, the above derivative is negative, meaning that the school can make a profitable deviation by lowering θ_0, a contradiction. Thus, $\theta_0 = \underline{\theta}$ in equilibrium. Then, the school's problem can be reduced to

$$\max_{z(\theta)} \int_{\underline{\theta}}^{\bar{\theta}} [\theta - G(z(\theta), \theta)] \, dF(\theta) - \underline{\theta}.$$

subject to $z(\theta)$ being increasing on $[\underline{\theta}, \bar{\theta}]$. Note that the integrand is decreasing in z. Thus, the school has an incentive to allocate as little of education as possible to the worker. Since the school cannot charge the worker for zero education, the "optimal" allocation is increasing and infinitesimal education. Formally, $z^o(\theta)$ is increasing on $[\underline{\theta}, \bar{\theta}]$ with $z^o(\underline{\theta}) = 0$; for $\forall\, \varepsilon > 0$, we have $z^o(\theta) < \varepsilon$. Under this allocation, social welfare is arbitrarily close to the first-best level, and the school's payoff is equal to social welfare minus arbitrarily small information rents and a positive rent $\underline{\theta}$ for participation.

We now turn to the unobserved case. Similarly, we assume that the labor market holds the worst belief for all positive off-path education levels. Since the wage schedule is independent of the actual tuition scheme, the school's problem can be similarly stated as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \{W(z) - G(z(\theta), \theta) - \mathbb{E}[\theta | \theta \in [\underline{\theta}, \theta_0)]\}\, dF(\theta).$$

subject to $z(\theta)$ being increasing on $[\theta_0, \bar{\theta}]$. In equilibrium, $W(z(\theta)) = \theta$, thus, by the same argument as in the observed case, we have $\theta_0 = \underline{\theta}$. Consequently, the analysis in Section 5 is completely applicable to this case. Specifically, the school-optimal separating equilibrium exists in the unobserved case, such that $z^u(\underline{\theta}) = 0$, $z^u(\theta)$ is increasing over $[\underline{\theta}, \bar{\theta}]$, and $z^u(\theta)$ satisfies the first-order condition

$$Q_\theta(z^u(\theta), \theta) \cdot \theta^{u\prime}(z^u(\theta)) - G_z(z^u(\theta), \theta) = 0.$$

Note that $z^u(\theta)$ is positive for $\forall\, \theta \in (\underline{\theta}, \bar{\theta}]$. Thus, $z^u(\theta) \geq z^o(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality for $\theta > \underline{\theta}$; that is, the worker obtains more education in the unobserved case than in the observed case, and thus, Theorem 5.1 still holds. Because education is unproductive, social welfare is unambiguously higher in the observed case. Indeed, by the definition in Section 4, the intensity of signaling is infinity for unproductive education, as the equilibrium education levels are positive in Spence's model (Spence 1973), but the fist-best education level is zero; thus, the observed case yields higher social welfare. Since education levels are higher in the unobserved case, the school leaves more information rents to the worker, and thus, the worker is better off in the unobserved case. But since social welfare is lower in the unobserved case, the school which is a residual claimant must be worse off than in the observed case.

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