Selling Signals

Zhuoran Lu

School of Management, Fudan University

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Motivation

In classic signaling models

- The sender's preference depends only on his intrinsic type.

In this paper

- The signaling cost also depends on a third party's choice.
- Education: the tuition scheme is chosen by the university.
- Luxury good: the price schedule is chosen by the retailer.
- Advertising: the tariff is chosen by the media company.

Key observation

- How receivers interpret and respond to the sender's signal depends on whether they observe the third party's choice.

Overview

Model

- *Seller* chooses a price schedule for a good with intrinsic value.
- *Buyer* chooses how much to purchase as a signal to *receivers*.

When receivers observe the price schedule

- Seller internalizes signaling when screening buyer's type.
- Buyer chooses a lower quantity than the first-best level.

When receivers do not observe the price schedule

- Receivers do not adjust their belief to the price schedule.
- Signal jamming effect leads to a more elastic demand.
- Seller charges lower prices and achieves lower profits.
- Buyer chooses a higher quantity and obtains higher utility.

Implications

Job market signaling

- Tuition transparency allows schools to commit to high tuition.
- Mandatory disclosure policies, such as U.S. Code § 1015a, may *unintentionally* raise education expenses and harm students.

Pricing strategies for signaling goods

- Profits are higher when the price is publicly observed.
- Luxury brands such as Louis Vuitton enjoy a reputation of never being on sale to maintain the brand's signaling value.
- The high costs of each year's Super Bowl commercials are widely reported, thereby enhancing the signaling value.

Literature

Signaling

- Spence (1973), Riley (1979), Milgrom and Roberts (1986), Bagwell and Bernheim (1996), Mailath and von Thadden (2013)

Screening

- Mussa and Rosen (1978), Maskin and Riley (1984), Rayo (2013)

Signal jamming

- Fudenberg and Tirole (1986), Holmstrom (1999), Zubrickas (2015)

Information intermediary

- Biglaiser (1993), Lizzeri (1999), Biglaiser and Li (2018)

Intermediate price transparency

- Inderst and Ottaviani (2012), Janssen and Shelegia (2015)

Model

Model

Players and actions

- A school chooses a tuition scheme $T : \mathbb{R}_+ \to \mathbb{R}_+$.
- A worker chooses education level z and pays $T(z)$.
  Ability $\theta \sim F(\theta)$ on $[\underline{\theta}, \bar{\theta}]$ with a positive density $f(\theta)$.
  Productivity $Q(z, \theta)$: $Q_z, Q_\theta > 0$ if $z > 0$; $Q(0, \theta) \equiv 0$.
- Competing employers offer wages $W(z) = \mathbb{E}[Q(z, \theta)]$.

Information

- Worker's ability θ is privately known.
- Education level z is publicly observed.
- In the *observed* case, employers observe T.
- In the *unobserved* case, employers do not observe T.

Payoffs

- School's expected profit $\Pi := \mathbb{E}[T(z(\theta))]$.
- Worker's utility $U(z,\theta) := W(z) - C(z,\theta) - T(z)$.

  Effort cost $C(z,\theta)$: $C_z, C_{zz} > 0$ if $z > 0$; $C(0,\theta) \equiv 0$.

  Single-crossing property: $C_{z\theta} < 0$.
- Worker has a zero-utility outside option.

First-best

- Social surplus $S(z,\theta) := Q(z,\theta) - C(z,\theta)$.
- $S(z,\theta)$ is strictly concave in z and $S_{z\theta} > 0$.
- The first-best $z^{fb}(\theta)$ is positive and increasing on $[\underline{\theta}, \bar{\theta}]$.

Perfect Bayesian Equilibrium

The observed case.

A profile $\{T^o, z^o(\theta;T), W^o(z;T)\}$ *and market belief is a PBE if*

(i) $z^o(\theta;T)$ *maximizes* $U(z,\theta)$ *for each* T *and* $W^o(z;T)$.

(ii) $W^o(z;T) = \mathbb{E}[Q(z,\theta)|T]$ *using Bayes' rule when possible.*

(iii) T^o *maximizes* $\Pi = \mathbb{E}[T(z^o(\theta;T))]$ *given* $z^o(\theta;T)$.

The unobserved case.

A profile $\{T^u, z^u(\theta;T), W^u\}$ *and market belief is a PBE if*

(i) $z^u(\theta;T)$ *maximizes* $U(z,\theta)$ *for each* T, *given* W^u.

(ii) $W^u(z) = \mathbb{E}[Q(z,\theta)|T^u]$ *using Bayes' rule when possible.*

(iii) T^u *maximizes* $\Pi = \mathbb{E}[T(z^u(\theta;T))]$ *given* $z^u(\theta;T)$.

Equilibrium Selection

Focus on continuous equilibria

- Equilibrium in which $z(\theta)$ is continuous if $z > 0$.

The observed case

- Focus on *seller-optimal equilibrium*.

The unobserved case

- Propose a new refinement, *Quasi-Divinity*

  $\Rightarrow$ *seller-optimal separating equilibrium* (Riley outcome).

Assumption 1.

$C_{zz\theta}(z,\theta) \leq 0$, $C_{z\theta\theta}(z,\theta) \geq 0$ and $h'(\theta) > -[h(\theta)]^2$ for all θ, where $h(\theta) := f(\theta)/[1-F(\theta)]$ is the hazard rate of $F(\theta)$.

A Direct Mechanism

Timing

- School proposes a contract $\{z(\theta), T(\theta)\}$ to the worker.
- Firms post $W(z)$ based on the observability of $\{z(\theta), T(\theta)\}$.
- Worker reports a type $\hat{\theta}$ to *only* the school.

Worker's problem

- Maximizes $U(\hat{\theta}, \theta) := W(z(\hat{\theta})) - C(z(\hat{\theta}), \theta) - T(\hat{\theta})$.
- $\{z(\theta), T(\theta)\}$ and $W(z)$ are *IC* if $\hat{\theta} = \theta$; *IR* if $U(\theta, \theta) \geq 0$.

School's problem

- In the observed case: Max. Π s.t. *IC*, *IR* and correct belief.
- In the unobserved case: Given $W(z)$, max. Π s.t. *IC* and *IR*.

Worker's Problem

Implementable allocation

- In both cases, let $U(\theta) \equiv U(\theta, \theta)$ be the equilibrium utility.
- An allocation $\{z(\theta), U(\theta)\}$ is *implementable* if *IC* and *IR*.

Lemma 1.
In each case, an allocation $\{z(\theta), U(\theta)\}$ *is implementable iff*

(i) $z(\theta)$ *is nondecreasing.*

(ii) *Define* $\theta_0 := \inf\{\theta | z(\theta) > 0\}$; *then, for any* $\theta \geq \theta_0$,

$$U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} -C_\theta(z(s), s)ds$$

subject to $U(\theta_0) \geq 0$.

School's Problem

For both cases

- *IC* means that $T(z(\theta)) = W(z(\theta)) - C(z(\theta), \theta) - U(\theta)$.
- Profit maximizing requires that $U(\theta_0) = 0$.
- Integrating by parts, the school's problem can be stated as

$$\max_{\{z,\theta_0\}} \int_{\theta_0}^{\bar{\theta}} \left[W(z) - C(z,\theta) + \frac{1 - F(\theta)}{f(\theta)} C_\theta(z,\theta) \right] dF(\theta)$$

$s.t.\ z(\theta)$ is nondecreasing.

The observed case

- By the law of total expectation, the school solves

$$\max_{\{z,\theta_0\}} \int_{\theta_0}^{\bar{\theta}} \left[S(z,\theta) + \frac{1-F(\theta)}{f(\theta)} C_\theta(z,\theta) \right] dF(\theta)$$

$s.t.$ $z(\theta)$ is nondecreasing.

The unobserved case

- Given some $W(z)$, the school solves

$$\max_{\{z,\theta_0\}} \int_{\theta_0}^{\bar{\theta}} \left[W(z) - C(z,\theta) + \frac{1-F(\theta)}{f(\theta)} C_\theta(z,\theta) \right] dF(\theta)$$

$s.t.$ $z(\theta)$ is nondecreasing.

- In equilibrium, $W(z) = \mathbb{E}[Q(z,\theta)]$ using Bayes' rule.

Spence's Signaling Game

Schools are competitive

- Bertrand competition leads to $T(z) \equiv 0$.

Equilibrium

- A strategy profile $\{z^s(\theta), W^s\}$ is a PBE if
  (i) Given W^s, $z^s(\theta)$ maximizes $U(z, \theta)$;
  (ii) $W^s(z) = \mathbb{E}[Q(z, \theta)]$ using Bayes' rule when possible.
- Focus on *least-cost separating equilibrium* (Riley outcome).

Over-Investment in Education

Proposition 0.

The least-cost separating equilibrium exists such that

(i) $z^s(\underline{\theta}) = z^{fb}(\underline{\theta})$; $z^s(\theta)$ *satisfies the first-order condition*

$$Q_z(z^s(\theta),\theta) + \underbrace{Q_\theta(z^s(\theta),\theta)\cdot\theta^{s\prime}(z^s(\theta))}_{\text{signaling effect}} - C_z(z^s(\theta),\theta) = 0;$$

(ii) $z^s(\theta)$ *is increasing on* $[\underline{\theta},\bar{\theta}]$; *thus* $W^s(z^s(\theta)) = Q(z^s(\theta),\theta)$.

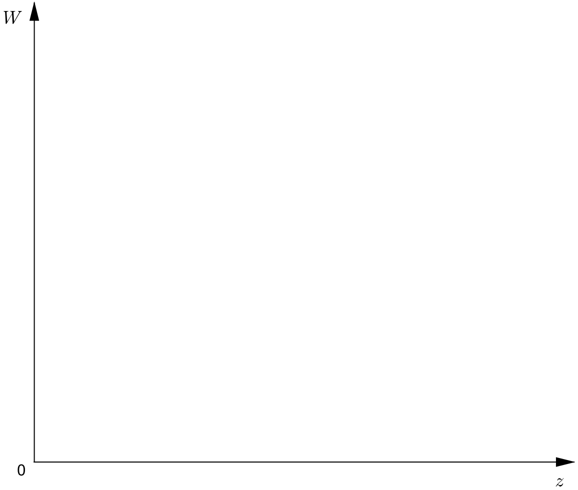
Corollary 0.

$z^s(\theta) \geq z^{fb}(\theta)$ *on* $[\underline{\theta},\bar{\theta}]$, *with strict inequality for* $\theta > \underline{\theta}$.

Example

A Binary-Type Example

Assumptions: $Q(z,\theta) = \theta z$, $C(z,\theta) = z^2/(2\theta)$, and $\theta \in \{\theta_L, \theta_H\}$.



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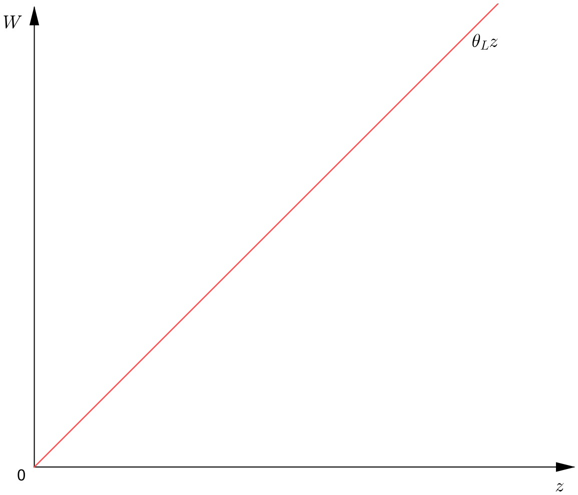

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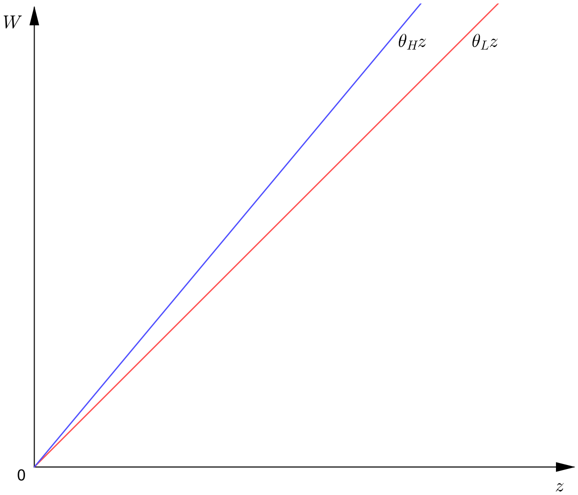
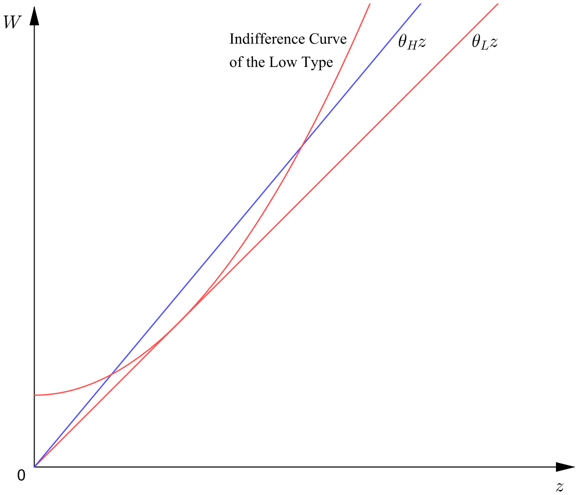


A Binary-Type Example

$$U_i = Q(z, \theta_i) - C(z, \theta_i),\ i = L, H.$$



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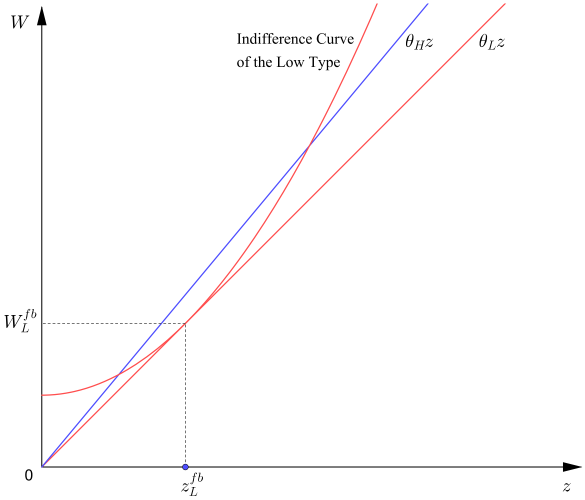

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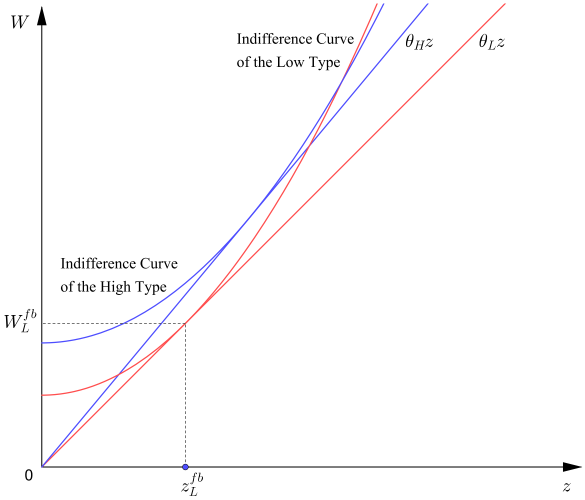

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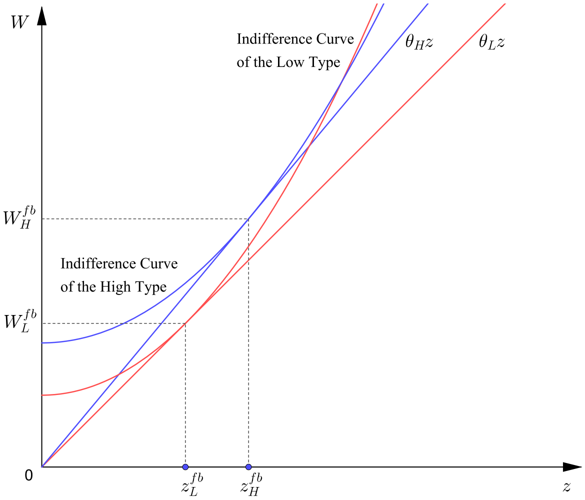
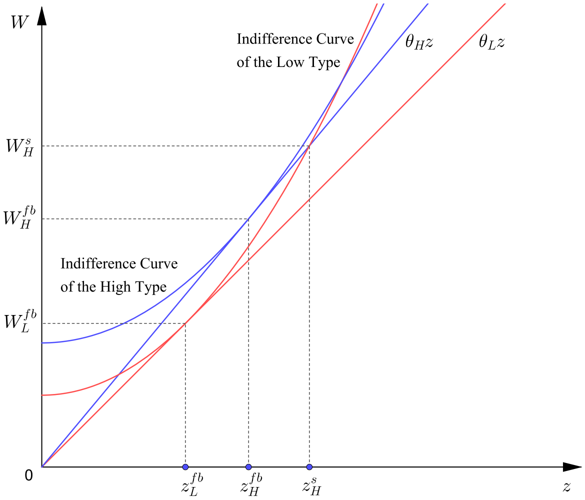


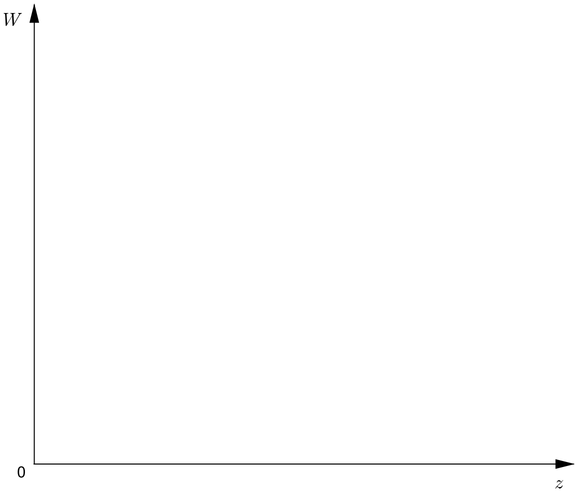
A Binary-Type Example

$$W_L^s - C(z_L^s, \theta_L) \geq W_H^s - C(z_H^s, \theta_L).$$



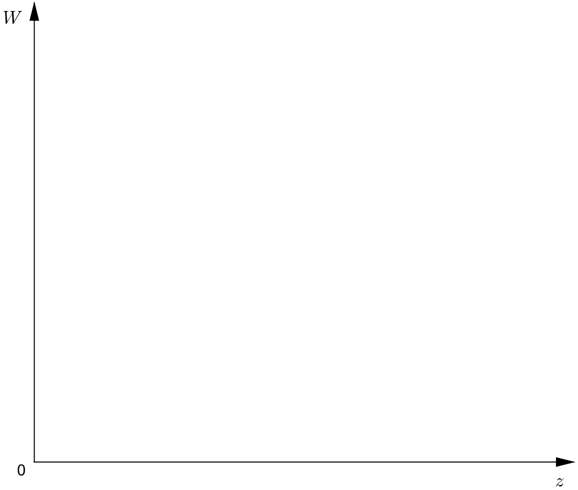
A Binary-Type Example

$$T_L + T_H = W_L - C(z_L, \theta_L) - U_L + W_H - C(z_H, \theta_H) - U_H.$$



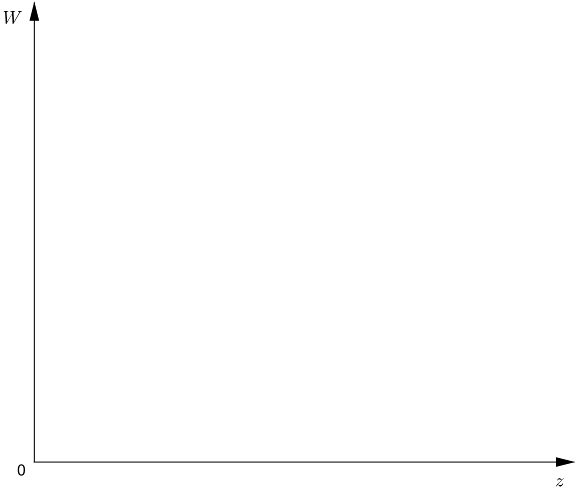
A Binary-Type Example

$$T_L + T_H = W_L - C(z_L, \theta_L) - 0 + W_H - C(z_H, \theta_H) - U_H.$$



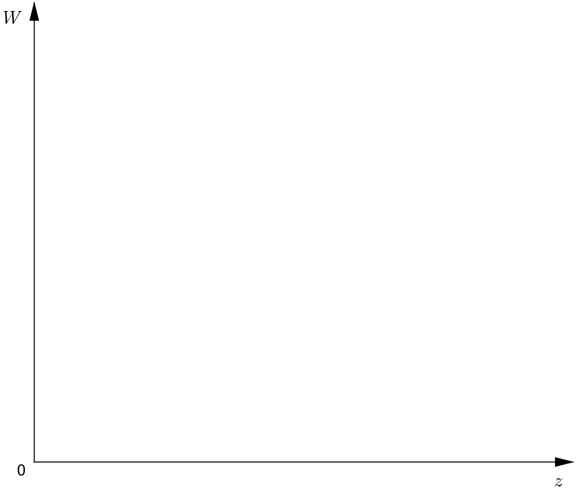
A Binary-Type Example

$$T_L + T_H = W_L - C(z_L, \theta_L) + W_H - C(z_H, \theta_H) - [C(z_L, \theta_L) - C(z_L, \theta_H)].$$



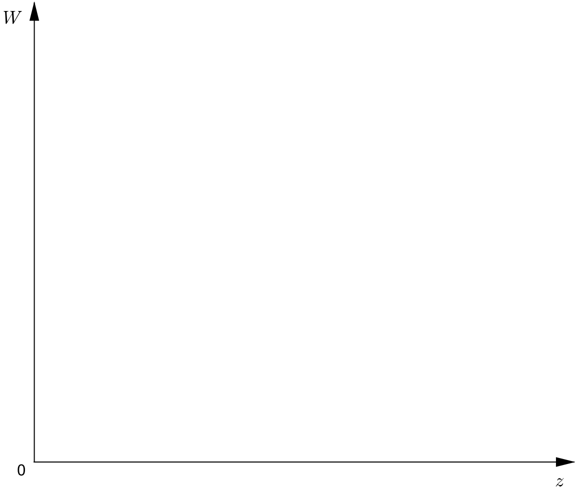
A Binary-Type Example

$$T_L + T_H = \boxed{W_L - C(z_L, \theta_L) - [C(z_L, \theta_L) - C(z_L, \theta_H)]} + W_H - C(z_H, \theta_H).$$



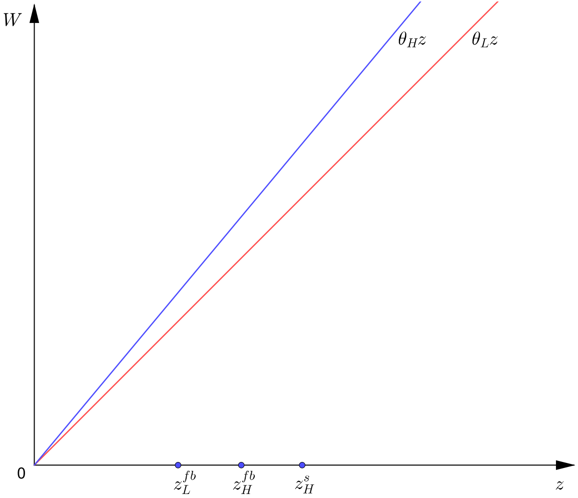
A Binary-Type Example

$$T_L + T_H = W_L - C(z_L, \theta_L) - [C(z_L, \theta_L) - C(z_L, \theta_H)] + \boxed{W_H - C(z_H, \theta_H)}.$$



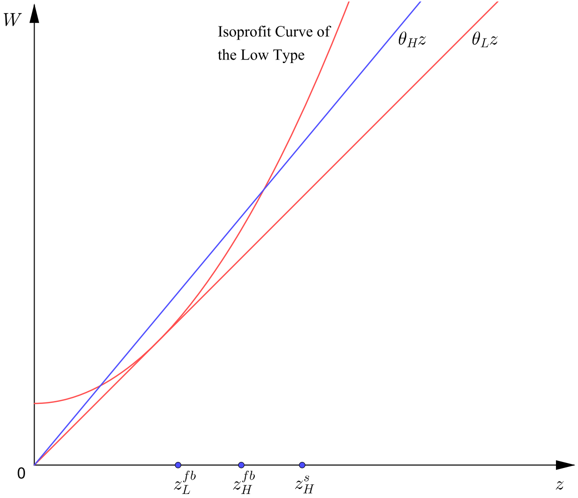
A Binary-Type Example

$$T_L^o + T_H^o = W_L^o - C(z_L^o, \theta_L) - [C(z_L^o, \theta_L) - C(z_L^o, \theta_H)] + W_H^o - C(z_H^o, \theta_H).$$



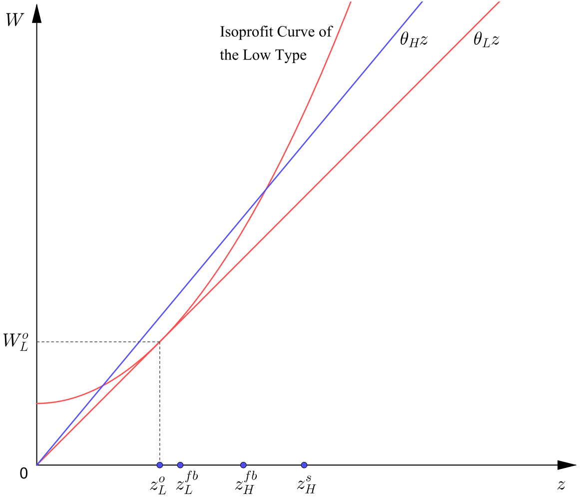
A Binary-Type Example

$$T_L^o + T_H^o = \boxed{W_L^o - C(z_L^o, \theta_L) - [C(z_L^o, \theta_L) - C(z_L^o, \theta_H)]} + W_H^o - C(z_H^o, \theta_H).$$



A Binary-Type Example

$$T_L^o + T_H^o = \boxed{W_L^o - C(z_L^o, \theta_L) - [C(z_L^o, \theta_L) - C(z_L^o, \theta_H)]} + W_H^o - C(z_H^o, \theta_H).$$



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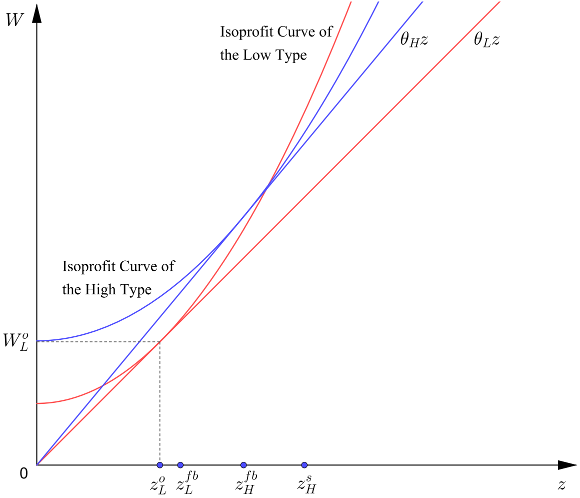

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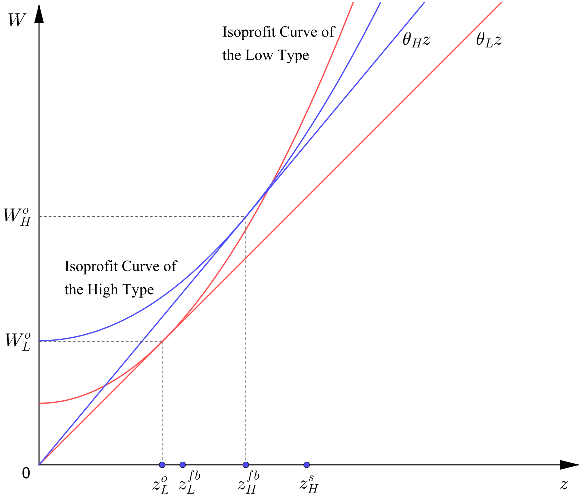
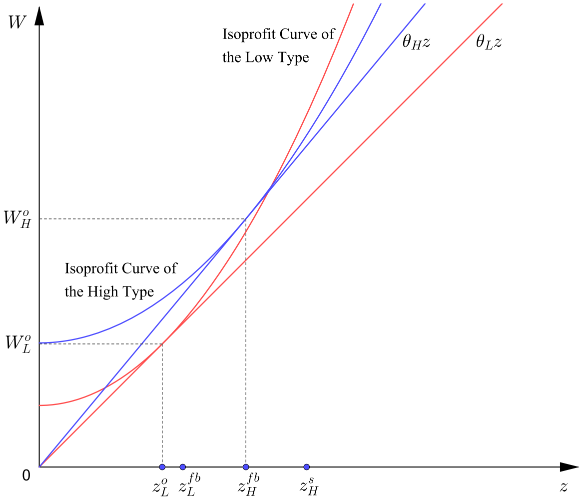


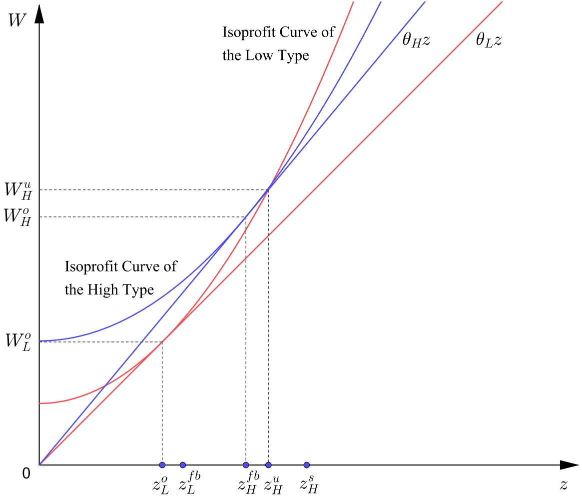
A Binary-Type Example

Alternative contract $\{(z', T')\}$: $z' = z_H^o$ and $T' = W_H^o - C(z_H^o, \theta_L) < T_H^o$.



A Binary-Type Example

$$W_L^u - C(z_L^u, \theta_L) - [C(z_L^u, \theta_L) - C(z_L^u, \theta_H)] \geq W_H^u - C(z_H^u, \theta_L) - [C(z_H^u, \theta_L) - C(z_H^u, \theta_H)].$$



The Observed Case

Solving the School's Problem

Marginal profit

- The *marginal profit* from selling to θ in the observed case is

$$MP^o(z,\theta) := S(z,\theta) - \left[-\frac{1-F(\theta)}{f(\theta)} C_\theta(z,\theta)\right].$$

- Following Toikka (2011), define

$$J(z,\theta) := \int_{\underline{\theta}}^{\theta} MP^o_z(z,s)ds.$$

- Let $I(z,\cdot) :=$ conv $J(z,\cdot)$ be the convex hull of $J(z,\cdot)$.
- Define the *generalized marginal profit* as

$$\overline{MP}^o(z,\theta) := MP^o(0,\theta) + \int_0^z I_\theta(x,\theta)dx.$$

Optimal allocation

- $\overline{MP}^o(z,\theta)$ is concave in z and has a unique maximizer $\bar{z}(\theta)$.
- The optimal allocation $z^o(\theta)$ is thus given by

$$z^o(\theta) = \begin{cases} \bar{z}(\theta) & \text{if } \theta \geq \theta_0^o, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Cutoff θ_0^o either solves $\overline{MP}^o(\bar{z}(\theta),\theta) = 0$, or is $\underline{\theta}$ otherwise.

Discontinuity in the price scheme

- $\bar{z}(\theta)$ exhibits *pooling* if $\bar{z}(\theta) \neq z^*(\theta) := \arg\max MP^o(z,\theta)$.
- W^o and thus T^o is discontinuous in such pooling intervals.

Under-Supply in Education

Proposition 1.

The seller-optimal equilibrium exists, such that on the equilibrium path, $z^o(\theta)$ is given by (1); W^o is given by $\mathbb{E}[Q(z^o(\theta),\theta)]$;

$$T^o(z^o(\theta)) = W^o(z^o(\theta)) - C(z^o(\theta),\theta) + \int_{\theta_0^o}^{\theta} C_\theta(z^o(s),s)ds.$$

Corollary 1.

$z^o(\theta) \leq z^{fb}(\theta)$ on $[\underline{\theta},\bar{\theta}]$, with strict inequality on $(\underline{\theta},\bar{\theta})$.

Idea

- Higher types benefit from cost advantage over lower types.
- School pays *information rents* to incentivize truth-telling.

Screening vs Signaling

Signaling induces over-education

- Signaling leads to "over-education" w.r.t. $T^o(z) + C(z, \theta)$.

Screening outweighs signaling

- The profit-maximizing tuition satisfies

$$T^{o\prime}(z) = Q_\theta(z, \theta^o(z)) \cdot \theta^{o\prime}(z) + \frac{1 - F(\theta^o(z))}{f(\theta^o(z))} \left[-C_{z\theta}(z, \theta^o(z)) \right].$$

- The welfare-maximizing tuition satisfies

$$T^{fb\prime}(z) = Q_\theta(z, \theta^{fb}(z)) \cdot \theta^{fb\prime}(z).$$

- The profit-maximizing tuition "over-taxes" signaling.

Mussa and Rosen's Screening Game

Firms observe worker's type

- Suppose worker reveals ability in school (e.g., by grades).
- Worker's reservation price for education becomes $S(z, \theta)$.
- School's marginal profit becomes

$$MP^{mr}(z, \theta) := S(z, \theta) - \frac{1 - F(\theta)}{f(\theta)} S_\theta(z, \theta).$$

- Given regularity, $z^{mr}(\theta)$ and θ_0^{mr} are solved analogously.

Signaling Mitigates Screening Distortion

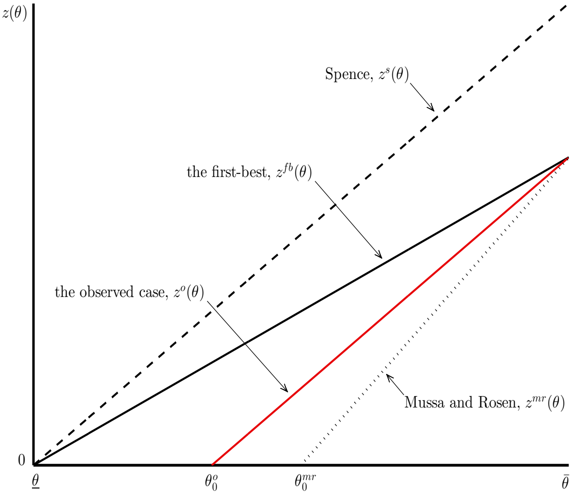
Proposition 2.

$z^o(\theta) \geq z^{mr}(\theta)$ on $[\theta_0^o, \bar{\theta}]$, with strict inequality for $\theta < \bar{\theta}$. Social welfare and the school's profit are higher in the observed case.

Idea

- Signaling reduces worker's incentive to mimic lower types.
- School pays lower information rents when signaling exists

$$\underbrace{\frac{1-F(\theta)}{f(\theta)}\left[-C_\theta(z,\theta)\right]}_{\text{information rents with signaling}} \leq \underbrace{\frac{1-F(\theta)}{f(\theta)} S_\theta(z,\theta)}_{\text{information rents without signaling}}$$



Assumptions: $Q(z, \theta) = \theta z + z$, $C(z, \theta) = z^2 + z - \theta z$, and $\theta \sim U[0, 1]$.

The Unobserved Case

Solving the School's Problem

- School's marginal profit

$$MP^u(z,\theta) := W(z) - \left[C(z,\theta) - \frac{1-F(\theta)}{f(\theta)}C_\theta(z,\theta)\right].$$

- Define school's *virtual cost*

$$G(z,\theta) := C(z,\theta) - \frac{1-F(\theta)}{f(\theta)}C_\theta(z,\theta).$$

- Single-crossing property: $G_{z\theta} < 0$.

Auxiliary signaling game

- Regard MP^u and G as worker's new utility and cost functions.
- It suffices to find an equilibrium with nondecreasing $z^u(\theta)$.

Equilibrium Existence

- For each $\theta \in [\underline{\theta}, \bar{\theta}]$, define $\Delta(z, \theta)$ as

$$\mathbb{E}\{Q(z,\theta)|\theta \in [\underline{\theta}, \bar{\theta}]\} - G(z,\theta) - \max_y \left\{Q(y,\underline{\theta}) - G(y,\theta)\right\}.$$

- Let $\underline{x}(\theta)$ and $\bar{x}(\theta)$ be the minimal and maximal roots of Δ.

Theorem 1.

(i) *There always exists a separating equilibrium in which $z^u(\theta)$ is increasing over $[\theta_0^u, \bar{\theta}]$ for some $\theta_0^u \geq \theta_0^o$, and satisfies that*

$$Q_z(z^u(\theta),\theta) + Q_\theta(z^u(\theta),\theta) \cdot \theta^{u\prime}(z^u(\theta)) - G_z(z^u(\theta),\theta) = 0.$$

(ii) *If $\bar{x}(\underline{\theta}) > \underline{x}(\bar{\theta})$, there also exists a pooling equilibrium in which $z^u(\theta) \equiv \tilde{z}$ on $[\underline{\theta}, \bar{\theta}]$ for some $\tilde{z} > 0$.*

(ii) *If $z^*(\underline{\theta}) > 0$ and $\bar{x}(\underline{\theta}) < \underline{x}(\bar{\theta})$, there only exists the seller-optimal separating equilibrium s.t. $(\theta_0^u, z^u(\theta_0^u)) = (\underline{\theta}, z^*(\underline{\theta}))$.*

Quasi-Divinity Refinement

Definition 1.
An equilibrium satisfies the Quasi-Divinity refinement if there *does not* exist an off-path signal $\hat{z} \in \mathbb{R}_+$, a receiver's response $\hat{w}$, and a positive-measure subset $\hat{\Theta} \subset [\underline{\theta}, \bar{\theta}]$ such that

(i) The following allocation of signals

$$z^d(\theta) = \begin{cases} \hat{z} & \text{if } \theta \in \hat{\Theta} \\ z^u(\theta) & \text{otherwise} \end{cases}$$

is nondecreasing in θ.

(ii) $\hat{w} - G(\hat{z}, \theta) > W^u(z^u(\theta)) - G(z^u(\theta), \theta)$ if and only if $\theta \in \hat{\Theta}$.

(iii) $\hat{w} < \mathbb{E}^{\hat{z}}[Q(\hat{z}, \theta) | \theta \in \hat{\Theta}]$, with the expectation formed under any quasi-divine belief, i.e., any receiver's posterior belief that has a distribution function $F^{\hat{z}}$ with $\text{supp}(F^{\hat{z}}) = \hat{\Theta}$.

Equilibrium Uniqueness

Theorem 2.
The unique equilibrium satisfying the Quasi-Divinity is the seller-optimal separating equilibrium s.t. $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^*(\theta_0^o))$.

Proof sketch

- For pooling equilibria, test $\hat{z}$ slightly higher than $\tilde{z}$.
- For other separating equilibria, test $\hat{z} \in (z^*(\theta_0^u), z^u(\theta_0^u))$.
- For SOSE, no such $\hat{z}$ for pessimistic quasi-divine beliefs.

Worker Obtains More Education

Theorem 3.
$z^u(\theta) \geq z^o(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality for $\theta > \theta_0^u$.

Idea

- Worker becomes more sensitive to tuition changes.
- The demand for education is therefore more elastic.
- School secretly cuts prices to *fool* the labor market.

School Achieves Lower Profits

Corollary 2.

In the unobserved case, the school's expected profit Π^u is strictly lower than its expected profit Π^o in the observed case.

Idea

- In equilibrium, firms have correct belief and offer lower wages.
- This reduces the worker's willingness to pay for education.

School Charges Lower Tuition

Proposition 3.

$T^o(z) > T^u(z)$ *on the common interval* $(z^*(\theta_0^o), z^*(\bar{\theta})]$.

Marginal Tuition

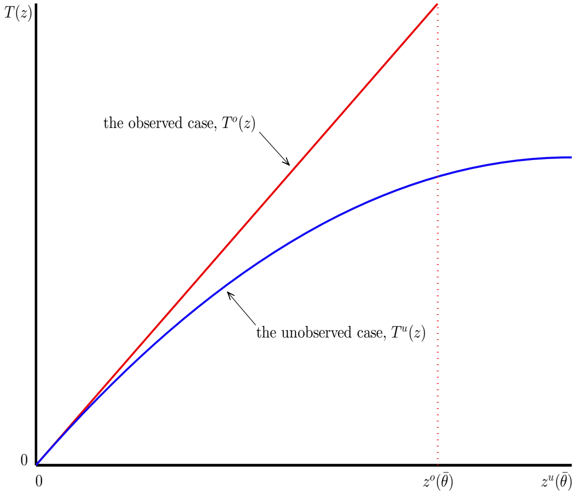
- In the observed case

$$T^{o\prime}(z) = Q_\theta(z, \theta^o(z)) \cdot \theta^{o\prime}(z) + \frac{1 - F(\theta^o(z))}{f(\theta^o(z))} \left[-C_{z\theta}(z, \theta^o(z)) \right].$$

- In the unobserved case

$$T^{u\prime}(z) = \frac{1 - F(\theta^u(z))}{f(\theta^u(z))} \left[-C_{z\theta}(z, \theta^u(z)) \right].$$

- School offers quantity discounts for higher education levels.



Assumptions: $Q(z, \theta) = \theta z + z$, $C(z, \theta) = z^2 + z - \theta z$, and $\theta \sim U[0, 1]$.

Worker Obtains Higher Utility

Proposition 4.

$U^o(\theta) \leq U^u(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality for $\theta > \theta_0^o$.

Idea

- In equilibrium, the market belief over tuition is correct.
- Tuition is lower in the unobserved case, benefiting worker.

Implication

- Tuition transparency allows schools to commit to high tuition.
- Mandatory disclosure policies, such as U.S. Code § 1015a, may *unintentionally* raise education expenses and harm students.

Under-Education and Over-Education Coexist

Proposition 5.

$z^u(\theta) \leq z^s(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality for $\theta > \underline{\theta}$.

Assumption 2.

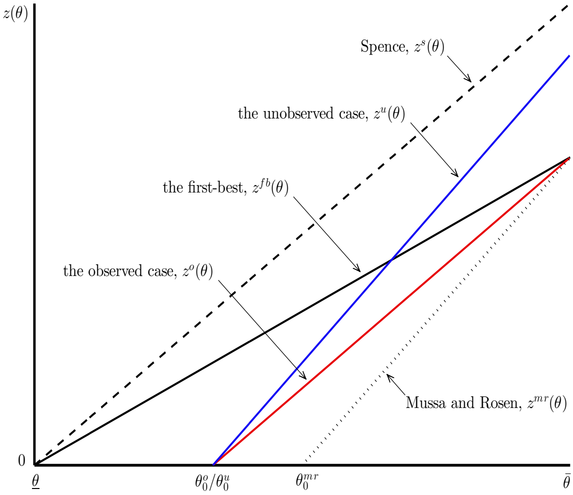
The function

$$Q_\theta(z^{fb}(\theta), \theta) \cdot \theta^{fb\prime}(z^{fb}(\theta)) + \frac{1 - F(\theta)}{f(\theta)} C_{z\theta}(z^{fb}(\theta), \theta) \qquad (*)$$

is single-crossing in θ, with a unique root $\theta^* \in (\underline{\theta}, \bar{\theta})$.

Proposition 6.

Suppose Assumption 2 holds, then there exists a type $\theta^w \in (\theta^*, \bar{\theta})$ such that $z^u(\theta) < z^{fb}(\theta)$ on $[\underline{\theta}, \theta^w)$ and $z^u(\theta) > z^{fb}(\theta)$ on $(\theta^w, \bar{\theta}]$.



Assumptions: $Q(z,\theta) = \theta z + z$, $C(z,\theta) = z^2 + z - \theta z$, and $\theta \sim U[0,1]$.

Welfare and Signaling Intensity

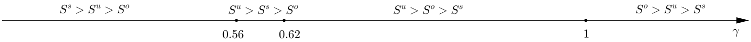
Example

- $Q(z,\theta) = \gamma\theta z + z$, $\gamma > 0$; $C(z,\theta) = z^2 + z - \theta z$; $\theta \sim U[0,1]$.
- $z^{fb}(\theta) = \frac{(\gamma+1)\theta}{2}$, $z^{s}(\theta) = \frac{(2\gamma+1)\theta}{2}$; Define signaling intensity:

$$\frac{z^s(\theta) - z^{fb}(\theta)}{z^{fb}(\theta)} = \frac{\gamma}{\gamma+1}.$$

  Thus, the greater γ is, the more intense signaling is.

- $z^{o}(\theta) = \frac{(\gamma+2)\theta-1}{2}$, $z^{u}(\theta) = (\gamma+1)\theta - \frac{\gamma+1}{\gamma+2}$; Cutoff $\theta^w = \frac{2}{\gamma+2}$.
- The welfare comparison among the three cases is given by



- Competition among sellers is not necessarily socially beneficial.

Extensions

Nonessential Education

Productivity

- $Q(0,\theta) \geq 0$ for all θ, and $Q_\theta(z,\theta) > 0$ for all z and θ.

The observed case

- The market is fully covered and has two segments:
  (i) Certification segment, i.e., $\theta \in [\underline{\theta}, \theta_0^o]$, in which $z^o(\theta) = 0$ and

$$T(0) = \mathbb{E}\{Q(z,\theta) | \theta \in [\underline{\theta}, \theta_0^o]\} = U(\theta_0^o).$$

  (ii) Education segment, i.e., $\theta \in (\theta_0^o, \bar{\theta}]$, in which

$$z^o(\theta) = \arg\max \overline{MP}^o(z,\theta).$$

The unobserved case

- $(\theta_0^u, z^u(\theta_0^u)) = (\underline{\theta}, z^*(\underline{\theta}))$, and $z^u(\theta) > z^o(\theta)$ for $\theta > \underline{\theta}$.

Unproductive Education

Productivity

- $Q_z(z,\theta) \equiv 0$, $Q(z,\theta) \geq 0$ and $Q_\theta(z,\theta) > 0$ for all z and θ.

The observed case

- There is only the certification segment, i.e., $z^o(\theta) \equiv 0$ and

$$T(0) = \mathbb{E}\{Q(z,\theta)|\theta \in [\underline{\theta}, \bar{\theta}]\}.$$

- Similar to Lizzeri (1999), but depends on specific belief.

The unobserved case

- $(\theta_0^u, z^u(\theta_0^u)) = (\underline{\theta}, 0)$, and $z^u(\theta)$ is increasing on $[\underline{\theta}, \bar{\theta}]$.
- Clearly, $z^u(\theta) > z^o(\theta)$ for $\theta > \underline{\theta}$.

Summary

We studied a signaling model in which

- Seller sets a price schedule for a good with intrinsic value.
- Buyer chooses how much to buy as a signal to receivers.
- Receivers *may* or *may not* observe the price schedule.

Main results

- In the observed case, buyer buys less than the first-best.
- In the unobserved case, buyer buys more than in the observed case; those of the highest types buy more than the first-best.

Application

Conspicuous Consumption

Players and actions

- A *retailer* chooses a price schedule $T : \mathbb{R}_+ \to \mathbb{R}_+$.
- A *consumer* chooses product quality z and pays $T(z)$.
  Wealth $\theta \sim F(\theta)$ on $[\underline{\theta}, \bar{\theta}]$ with a positive density $f(\theta)$.
  Externality $Q(z, \theta)$: $Q_z, Q_\theta > 0$ if $z > 0$; $Q(0, \theta) \equiv 0$.
- Competing *social contacts* offer rewards $W(z) = \mathbb{E}[Q(z, \theta)]$.

Information

- Consumer's wealth θ is privately known.
- Product quality z is publicly observed.
- In the observed case, social contacts observe T.
- In the unobserved case, social contacts do not observe T.

Payoffs

- Retailer's expected profit $\Pi := \mathbb{E}[T(z(\theta)) - C(z(\theta))]$.
- Consumer's utility $U(z, \theta) := W(z) + V(z, \theta) - T(z)$.
  Intrinsic value $V(z, \theta)$: $V_z > 0$ if $z > 0$; $V(0, \theta) \equiv 0$.
  Single-crossing property: $V_{z\theta} > 0$.
- Consumer has a zero-utility outside option.

First-best

- Social surplus $S(z, \theta) := Q(z, \theta) + V(z, \theta) - C(z)$.
- $S(z, \theta)$ is strictly concave in z and $S_{z\theta} > 0$.
- The first-best $z^{fb}(\theta)$ is positive and increasing on $[\underline{\theta}, \bar{\theta}]$.

Advertising

Players and actions

- A *media company* chooses a price schedule $T : \mathbb{R}_+ \to \mathbb{R}_+$.
- A *producer* chooses advertising level z and pays $T(z)$.
  Quality $\theta \sim F(\theta)$ on $[\underline{\theta}, \bar{\theta}]$ with a positive density $f(\theta)$.
  Demand in one time period $D(z)$: $D_z > 0$; $D(0) = 0$.
- *Consumers* pay $\mathbb{E}[\theta]$ in the first purchase, and θ in the second.

Information

- Quality θ will not reveal until purchase.
- Advertising level z is publicly observed.
- In the observed case, consumers observe T.
- In the unobserved case, consumers do not observe T.

Payoffs

- Media company's expected profit $\Pi := \mathbb{E}[T(z(\theta)) - C(z(\theta))]$.
- Producer's net payoff

$$U(z,\theta) := (\mathbb{E}[\theta|z] + \theta)D(z) - T(z).$$

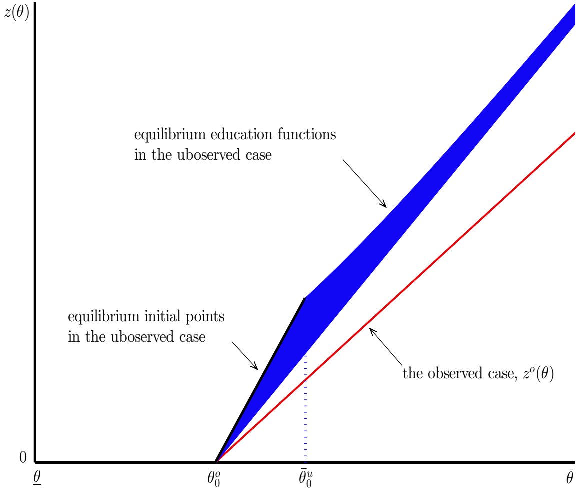
Single-crossing property: $U_{z\theta} > 0$.

- Producer has a zero-utility outside option.

First-best

- Social surplus $S(z,\theta) := 2\theta D(z) - C(z)$.
- $S(z,\theta)$ is strictly concave in z and $S_{z\theta} > 0$.
- The first-best $z^{fb}(\theta)$ is positive and increasing on $[\underline{\theta}, \bar{\theta}]$.

Appendix: Separating Equilibria of the Unobserved Case



Assumptions: $Q(z,\theta) = \theta z + z$, $C(z,\theta) = z^2 + z - \theta z$, and $\theta \sim U[0,1]$.