

Selling Signals

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October 13, 2018

Motivation

In classic signaling models

- ▶ The sender's preference depends only on his intrinsic type.

In this paper

- ▶ The signaling cost also depends on a third party's choice.
- ▶ Education: the tuition scheme is chosen by the university.
- ▶ Luxury good: the price schedule is chosen by the retailer.
- ▶ Advertising: the tariff is chosen by the media company.

Key observation

- ▶ How receivers interpret and respond to the sender's signal depends on whether they observe the third party's choice.

Overview

Model

- ▶ *Principal* sets a price schedule for a good with intrinsic value.
- ▶ *Agent* chooses how much to purchase as a signal to *market*.

When market observes the price schedule

- ▶ Principal internalizes signaling when screening agent's type.
- ▶ Agent chooses a lower quantity than the first-best level.

When market does not observe the price schedule

- ▶ Market does not adjust its belief to the price schedule.
- ▶ Signal jamming effect leads to a more elastic demand.
- ▶ Principal charges lower prices and achieves lower profits.
- ▶ Agent chooses a higher quantity and obtains higher utility.

Implications

Job market signaling

- ▶ Tuition transparency allows schools to commit to high tuition.
- ▶ Mandatory disclosure policies, such as U.S. Code § 1015a, may *unintentionally* raise education expenses and harm students.

Pricing strategies for signaling goods

- ▶ Profits are higher when the price is publicly observed.
- ▶ Luxury brands such as Louis Vuitton enjoy a reputation of never being on sale to maintain the brand's signaling value.
- ▶ The high costs of each year's Super Bowl commercials are widely reported, thereby enhancing the signaling value.

Literature

Signaling

- ▶ Spence (1973), Riley (1979), Milgrom & Roberts (1986), Mailath (1987), Bagwell & Riordan (1991), Ireland (1994), Bagwell & Bernheim (1996), Andersson (1996), Waldman (2016).

Screening

- ▶ Mussa & Rosen (1978), Maskin & Riley (1984), Guesnerie & Laffont (1984), Calzolari & Pavan (2006), Rayo (2013).

Signal jamming

- ▶ Fudenberg & Tirole (1986), Holmstrom (1999), Chan et al. (2007).

Intermediate price transparency

- ▶ Inderst & Ottaviani (2012), Janssen & Shelegia (2015).

MODEL

Model

Players and actions

- ▶ A school chooses a tuition scheme $T(z) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$.
- ▶ A worker chooses education level z and pays $T(z)$.
Ability $\theta \sim F(\theta)$ on $[\underline{\theta}, \bar{\theta}]$ with a positive density $f(\theta)$.
Productivity $Q(z, \theta)$: $Q_z, Q_\theta > 0$ if $z > 0$; $Q(0, \theta) \equiv 0$.
- ▶ Multiple competing firms offer wages $W(z) = \mathbb{E}[Q(z, \theta)|z]$.

Information

- ▶ Worker's ability θ is privately known.
- ▶ Education level z is publicly observed.
- ▶ In the *observed* case, firms observe $T(z)$.
- ▶ In the *unobserved* case, firms do not observe $T(z)$.

Payoffs

- ▶ School's expected profit $\Pi := \mathbb{E}[T(z(\theta))]$.
- ▶ Worker's utility $U(z, \theta) := W(z) - T(z) - C(z, \theta)$.
Effort cost $C(z, \theta)$: $C_z, C_{zz} > 0$ if $z > 0$; $C(0, \theta) \equiv 0$.
Single-crossing property: $C_{z\theta} < 0$ if $z > 0$.
- ▶ Worker has a zero-utility outside option.

First-best

- ▶ Social surplus $S(z, \theta) := Q(z, \theta) - C(z, \theta)$.
- ▶ $S(z, \theta)$ is strictly quasi-concave in z and $S_{z\theta}(z, \theta) > 0$.
- ▶ The first-best $z^{fb}(\theta)$ is positive and increasing on $[\underline{\theta}, \bar{\theta}]$.

Perfect Bayesian Equilibrium

The observed case

A strategy profile $\{T^o, z^o(\theta; T), W^o(z; T)\}$ is a PBE if

- ▶ $z^o(\theta; T)$ maximizes $U(z, \theta)$ for each T and $W^o(z; T)$.
- ▶ $W^o(z; T) = \mathbb{E}[Q(z, \theta) | z^o(\theta; T)]$ using Bayes' rule.
- ▶ T^o maximizes $\Pi = \mathbb{E}[T(z^o(\theta; T))]$ given $z^o(\theta; T)$.

The unobserved case

A strategy profile $\{T^u, z^u(\theta; T), W^u(z; T^u)\}$ is a PBE if

- ▶ $z^u(\theta; T)$ maximizes $U(z, \theta)$ for each T , given $W^u(z; T^u)$.
- ▶ $W^u(z; T^u) = \mathbb{E}[Q(z, \theta) | z^u(\theta; T^u)]$ using Bayes' rule.
- ▶ T^u maximizes $\Pi = \mathbb{E}[T(z^u(\theta; T))]$ given $z^u(\theta; T)$.

Equilibrium Selection

School-optimal separating equilibrium

- ▶ Equilibrium with the highest Π s.t. $z(\theta)$ is onto if $z(\theta) > 0$.

Idea

- ▶ In Spence's, D1 refinement selects the *least-cost separating equilibrium* (LCSE) such that $z(\theta)$ is onto and $z(\underline{\theta}) = z^{fb}(\underline{\theta})$.
- ▶ Given equilibrium T , the continuation game constitutes LCSE.
- ▶ In a discrete-type version of the model, a pooling equilibrium does not exist whenever the highest type is sufficiently likely.

Regularity conditions

- ▶ $C_{z\theta\theta}(z, \theta) \geq 0$ and $F(\theta)$ has a non-decreasing hazard rate.

A Direct Mechanism

Timing

- ▶ School proposes a contract $\langle z(\theta), T(z) \rangle$ to the worker.
- ▶ Firms post $W(z)$ based on the observability of $\langle z(\theta), T(z) \rangle$.
- ▶ Worker reports a type $\hat{\theta}$ to only the school.

Worker's problem

- ▶ Maximizes $U(\hat{\theta}, \theta) := W(z(\hat{\theta})) - T(z(\hat{\theta})) - C(z(\hat{\theta}), \theta)$.
- ▶ $\{\langle z(\theta), T(z) \rangle, W(z)\}$ is *IC* if $\hat{\theta} = \theta$; *IR* if $U(\theta, \theta) \geq 0$.

School's problem

- ▶ In the observed case: max. Π s.t. *IC*, *IR* and correct belief.
- ▶ In the unobserved case: given $W(z)$, max. Π s.t. *IC* and *IR*.

Worker's Problem

Implementable allocation

- ▶ In both cases, let $U(\theta) \equiv U(\theta, \theta)$ be the equilibrium utility.
- ▶ An allocation $\langle z(\theta), U(\theta) \rangle$ is *implementable* if *IC* and *IR*.

Lemma 1.

In each case, an allocation $\langle z(\theta), U(\theta) \rangle$ is implementable iff

- $z(\theta)$ is non-decreasing.
- Define $\theta_0 := \inf\{\theta | z(\theta) > 0\}$, then for all $\theta \geq \theta_0$,

$$U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} -C_{\theta}(z(s), s) ds \geq 0.$$

School's Problem

For both cases

- ▶ *IC* means that $T(z(\theta)) = W(z(\theta)) - C(z(\theta), \theta) - U(\theta)$.
- ▶ Profit maximizing requires that $U(\theta_0) = 0$.
- ▶ Integrating by parts, the school's problem can be stated as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ W(z) - C(z, \theta) + \frac{1 - F(\theta)}{f(\theta)} C_{\theta}(z, \theta) \right\} dF(\theta)$$

s.t. $z(\theta)$ non-decreasing.

The observed case

- ▶ By law of total expectation, the school solves

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ S(z, \theta) + \frac{1 - F(\theta)}{f(\theta)} C_{\theta}(z, \theta) \right\} dF(\theta)$$

s.t. $z(\theta)$ non-decreasing.

The unobserved case

- ▶ Given some $W(z)$, the school solves

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ W(z) - C(z, \theta) + \frac{1 - F(\theta)}{f(\theta)} C_{\theta}(z, \theta) \right\} dF(\theta)$$

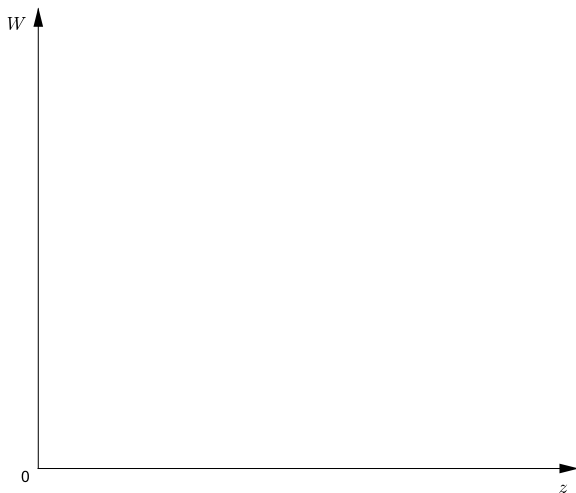
s.t. $z(\theta)$ non-decreasing.

- ▶ In equilibrium, $W(z) = \mathbb{E}[Q(z, \theta) | z(\theta)]$.

EXAMPLE

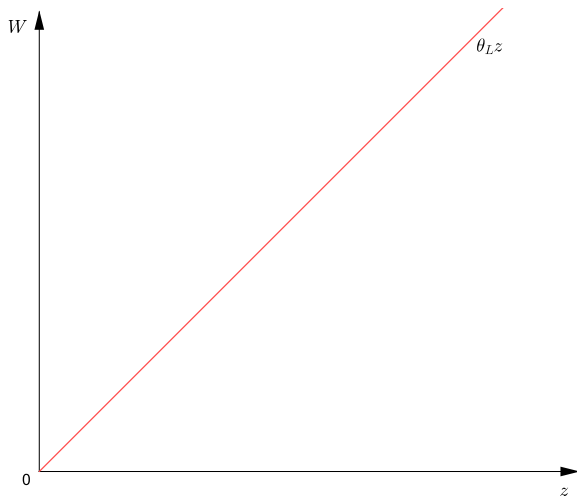
A Binary-Type Example

Assumptions: $Q(z, \theta) = \theta z$, $C(z, \theta) = z^2/(2\theta)$, and $\theta \in \{\theta_L, \theta_H\}$.



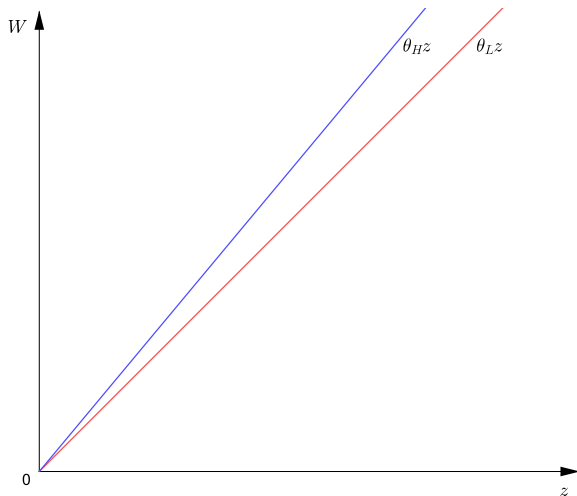
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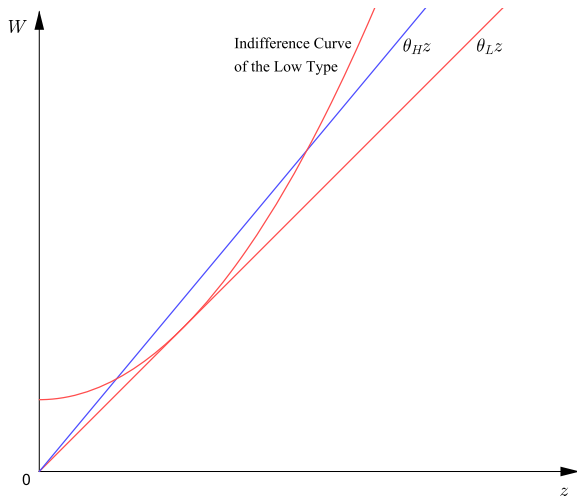
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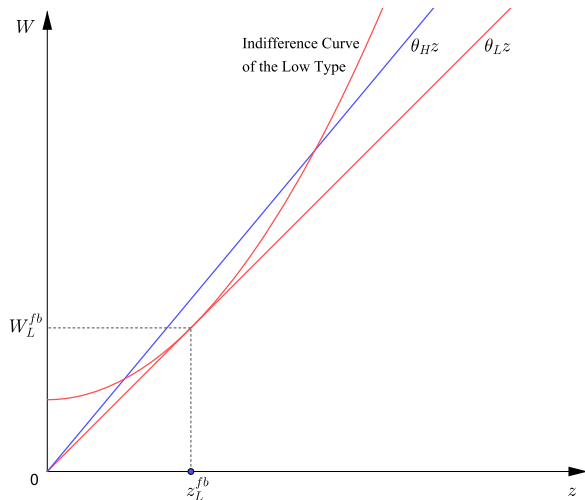
A Binary-Type Example

$$U_i = Q(z, \theta_i) - C(z, \theta_i), \quad i = L, H.$$



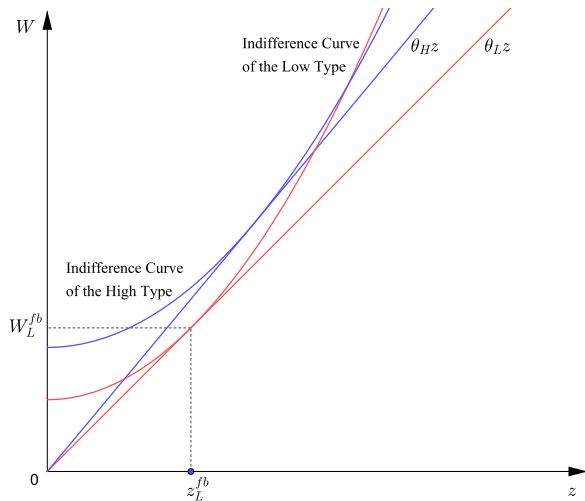
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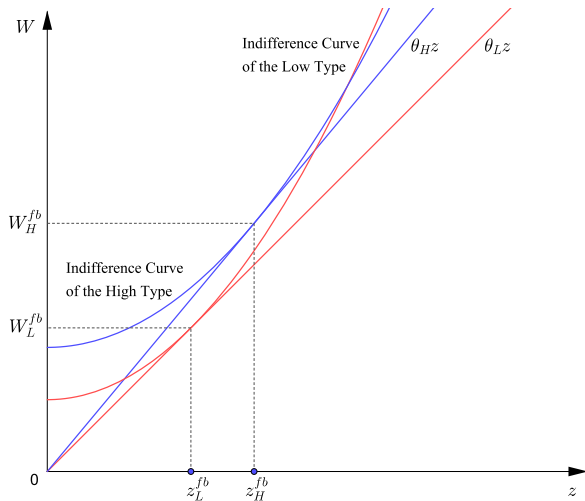
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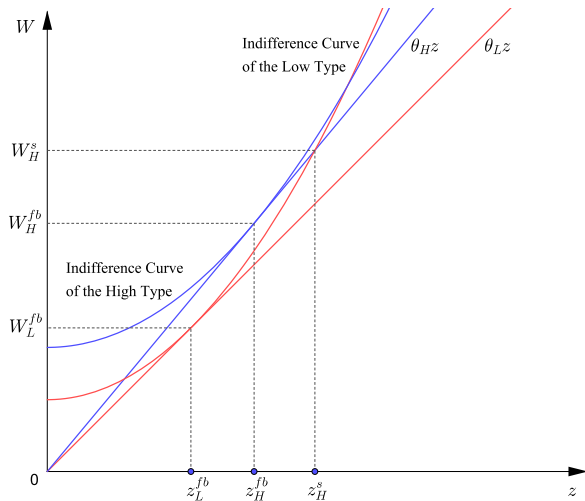
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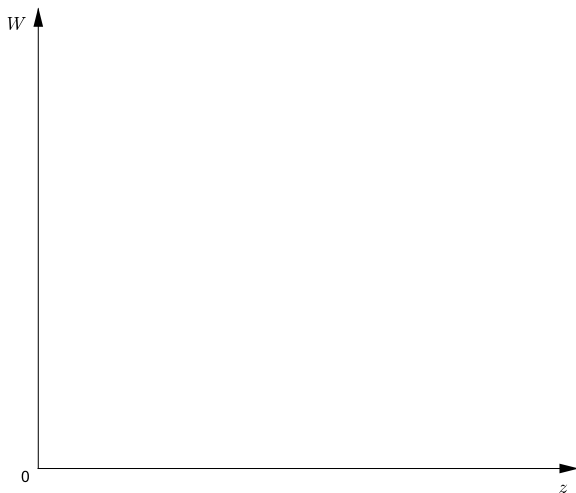
A Binary-Type Example

$$W_L^s - C(z_L^s, \theta_L) \geq W_H^s - C(z_H^s, \theta_L).$$



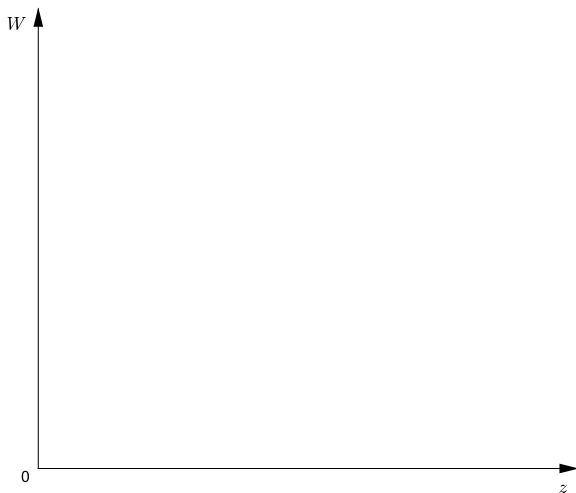
A Binary-Type Example

$$T_L + T_H = W_L - C(z_L, \theta_L) - U_L + W_H - C(z_H, \theta_H) - U_H.$$



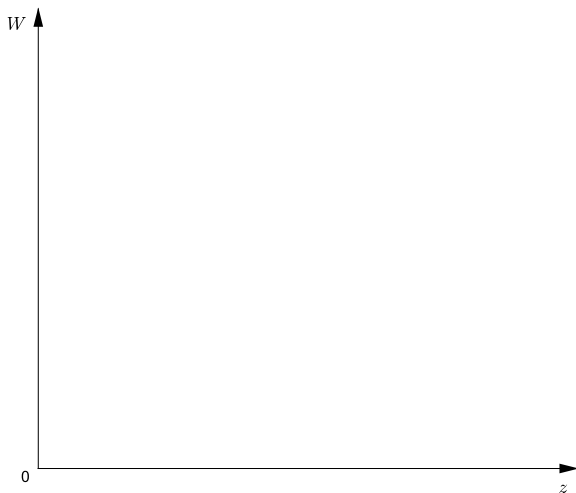
A Binary-Type Example

$$T_L + T_H = W_L - C(z_L, \theta_L) - 0 + W_H - C(z_H, \theta_H) - U_H.$$



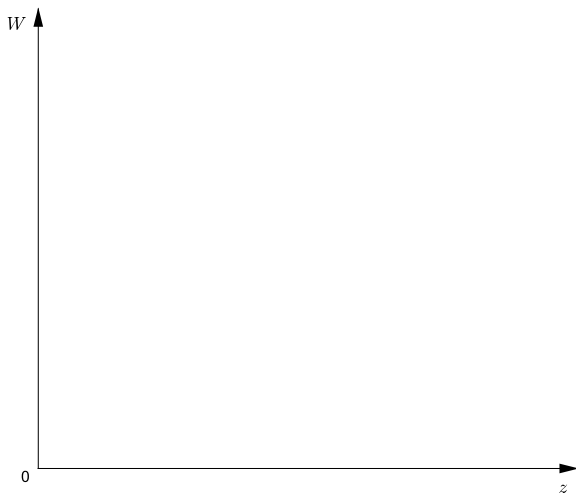
A Binary-Type Example

$$T_L + T_H = W_L - C(z_L, \theta_L) + W_H - C(z_H, \theta_H) - [C(z_L, \theta_L) - C(z_L, \theta_H)].$$



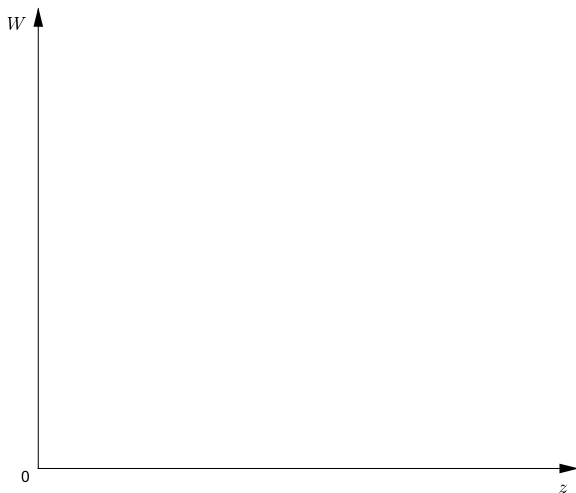
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$$T_L + T_H = \boxed{W_L - C(z_L, \theta_L) - [C(z_L, \theta_L) - C(z_L, \theta_H)]} + W_H - C(z_H, \theta_H).$$



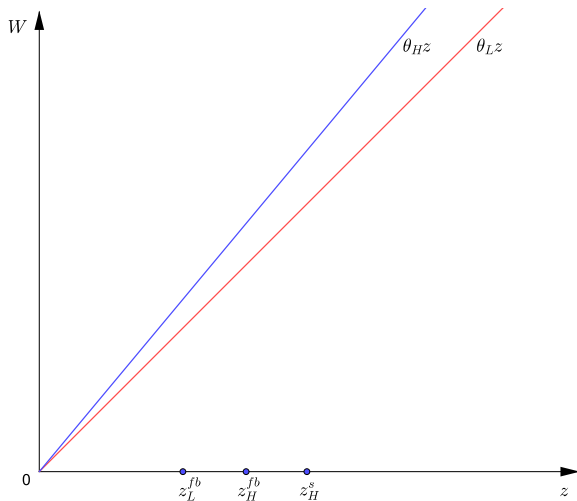
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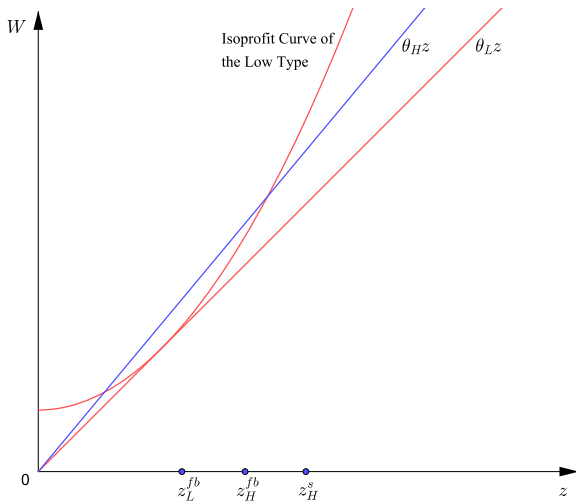
A Binary-Type Example

$$T_L^o + T_H^o = W_L^o - C(z_L^o, \theta_L) - [C(z_L^o, \theta_L) - C(z_L^o, \theta_H)] + W_H^o - C(z_H^o, \theta_H).$$



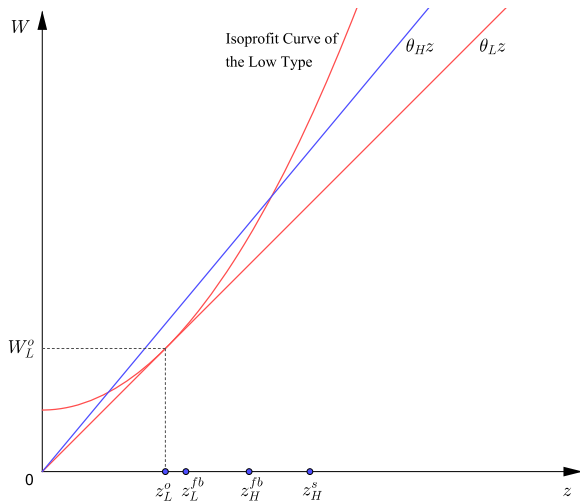
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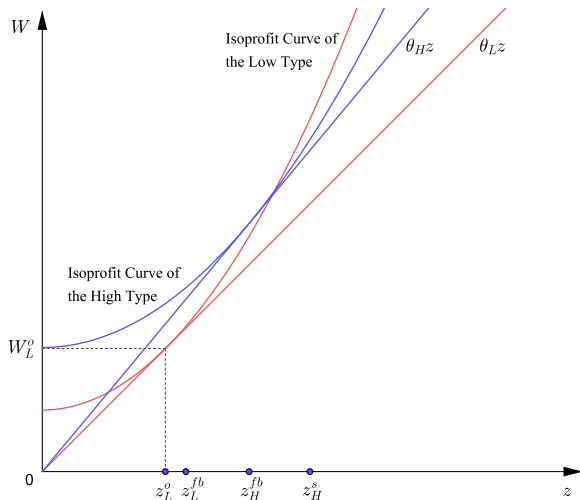
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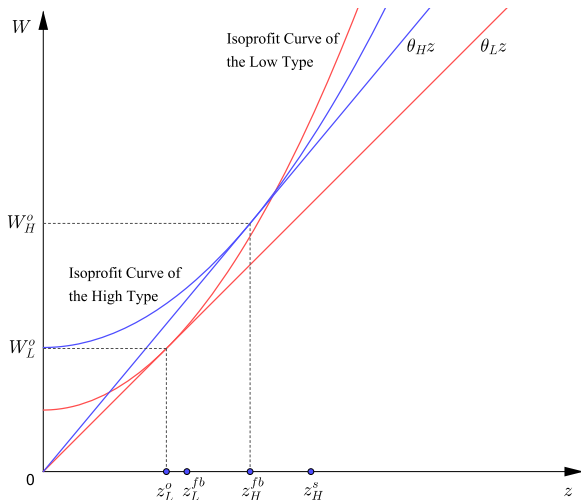
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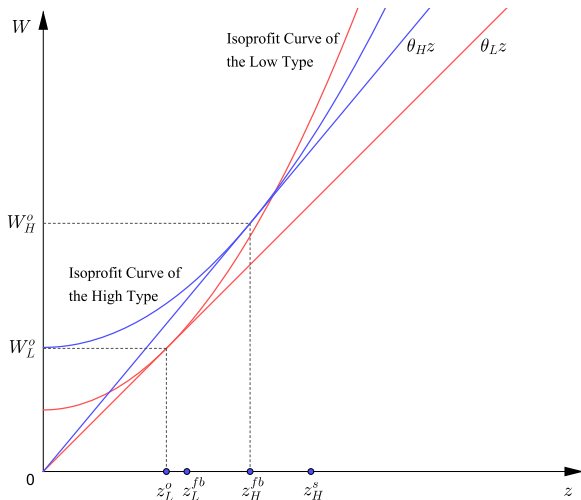
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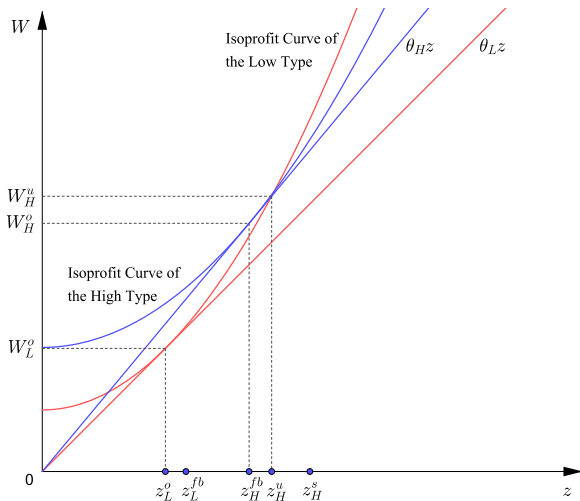
A Binary-Type Example

Alternative contract $\{(z', T')\}$: $z' = z_H^o$ and $T' = W_H^o - C(z_H^o, \theta_L) < T_H^o$.



A Binary-Type Example

$$W_L^u - C(z_L^u, \theta_L) - [C(z_L^u, \theta_L) - C(z_L^u, \theta_H)] \geq W_H^u - C(z_H^u, \theta_L) - [C(z_H^u, \theta_L) - C(z_H^u, \theta_H)].$$



THE OBSERVED CASE

Solving the School's Problem

Marginal profit

- ▶ The *marginal profit* from selling to θ in the observed case is

$$MP^o(z, \theta) := S(z, \theta) + \frac{1 - F(\theta)}{f(\theta)} C_\theta(z, \theta).$$

- ▶ $MP^o(z, \theta)$ has a unique maximizer $z^*(\theta)$ which is increasing.

Optimal allocation

- ▶ The optimal allocation $z^o(\theta)$ is given by

$$z^o(\theta) = \begin{cases} z^*(\theta) & \text{if } \theta \geq \theta_0^o, \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Cutoff θ_0^o either solves $MP^o(z^*(\theta), \theta) = 0$, or is $\underline{\theta}$ otherwise.

Under-Supply in Education

Proposition 1.

The school-optimal separating equilibrium exists, such that on the equilibrium path, $z^o(\theta)$ is given by (1); $W^o(z)$ equals $Q(z, \theta^o(z))$;

$$T^o(z^o(\theta)) = S(z, \theta^o(z)) + \int_{\theta_0^o}^{\bar{\theta}} C_{\theta}(z, \theta^o(z)) d\theta.$$

Corollary 1.

$z^o(\theta) \leq z^{fb}(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality for $\theta < \bar{\theta}$.

Idea

- ▶ Higher types benefit from cost advantage over lower types.
- ▶ School pays *information rents* to incentivize truth-telling.

Screening vs Signaling

Signaling induces over-education

- ▶ Signaling leads to “over-education” w.r.t. $T^o(z) + C(z, \theta)$.

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Screening outweighs signaling

- ▶ The profit-maximizing tuition satisfies

$$T^{o'}(z) = Q_{\theta}(z, \theta^o(z)) \cdot \theta^{o'}(z) + \frac{1 - F(\theta^o(z))}{f(\theta^o(z))} [-C_{z\theta}(z, \theta^o(z))].$$

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- ▶ The welfare-maximizing tuition satisfies

$$T^{fb'}(z) = Q_{\theta}(z, \theta^{fb}(z)) \cdot \theta^{s'}(z).$$

Screening vs Signaling

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- ▶ The profit-maximizing tuition “over-taxes” signaling.

Mussa and Rosen's Screening Game

Firms observe worker's type

- ▶ Suppose worker reveals ability in school (e.g., by grades).
- ▶ Worker's intrinsic value for education becomes $S(z, \theta)$.
- ▶ School's marginal profit becomes

$$MP^{mr}(z, \theta) := S(z, \theta) - \frac{1 - F(\theta)}{f(\theta)} S_{\theta}(z, \theta).$$

- ▶ Given regularity, $z^{mr}(\theta)$ and θ_0^{mr} are solved analogously.

Signaling Mitigates Screening Distortion

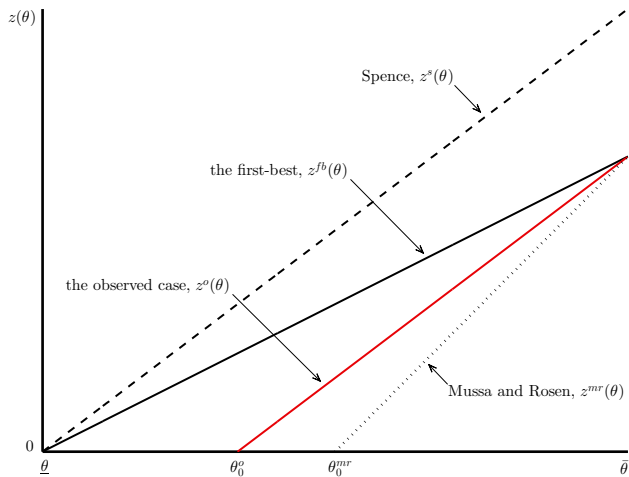
Proposition 2.

$z^o(\theta) \geq z^{mr}(\theta)$ on $[\theta_0^o, \bar{\theta}]$, with strict inequality for $\theta < \bar{\theta}$. Social welfare and the school's profit are higher in the observed case.

Idea

- ▶ Signaling reduces worker's incentive to mimic lower types.
- ▶ School pays lower information rents when signaling exists:

$$\underbrace{\frac{1 - F(\theta)}{f(\theta)} [-C_\theta(z, \theta)]}_{\text{information rents with signaling}} \leq \underbrace{\frac{1 - F(\theta)}{f(\theta)} S_\theta(z, \theta)}_{\text{information rents without signaling}}$$



Assumptions: $Q(z, \theta) = \theta z + z$, $C(z, \theta) = z^2 + z - \theta z$, and $\theta \sim U[0, 1]$.

THE UNOBSERVED CASE

Solving the School's Problem

- ▶ School's marginal profit

$$MP^u(z, \theta) := W(z) - C(z, \theta) + \frac{1 - F(\theta)}{f(\theta)} C_\theta(z, \theta).$$

- ▶ Define school's *virtual cost*

$$G(z, \theta) := C(z, \theta) - \frac{1 - F(\theta)}{f(\theta)} C_\theta(z, \theta).$$

Auxiliary signaling game

- ▶ Regard MP^u and G as worker's new utility and cost functions.
- ▶ It suffices to find a separating equilibrium $\{z^u(\theta), W^u(z)\}$.

Characterizing the Equilibrium

Proposition 3.

The school-optimal separating equilibrium exists such that

- (i) $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$.
- (ii) $z^u(\theta)$ is increasing over $[\theta_0^u, \bar{\theta}]$ and satisfies

$$Q_z(z^u(\theta), \theta) + Q_\theta(z^u(\theta), \theta) \cdot \theta^{u'}(z^u(\theta)) - G_z(z^u(\theta), \theta) = 0.$$

- (iii) $W^u(z^u(\theta)) = Q(z^u(\theta), \theta)$ for all $\theta \in [\theta_0^u, \bar{\theta}]$.

Worker Obtains More Education

Theorem 1.

$z^u(\theta) \geq z^o(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality for $\theta > \theta_0^u$.

Proof

- ▶ In the observed case

$$MP_z^o(z, \theta) = Q_z(z, \theta) - G_z(z, \theta).$$

- ▶ In the unobserved case

$$MP_z^u(z, \theta) = Q_z(z, \theta) + \underbrace{Q_\theta(z, \theta) \cdot \theta^{u'}(z)}_{\text{signal jamming effect}} - G_z(z, \theta).$$

- ▶ Since $z^u(\theta)$ is increasing, $MP_z^u(z, \theta) \geq MP_z^o(z, \theta)$.
- ▶ Since $MP^o(z, \theta)$ is strictly quasi-concave in z , $z^u(\theta) \geq z^o(\theta)$.

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Idea

- ▶ Worker becomes more sensitive to tuition changes.
- ▶ School secretly cuts prices to *fool* the labor market.

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Empirics

- ▶ According to the reports by The College Board:

From 2007-08 through 2010-11, the percentage of institutional grant aid that meet students' financial need at private nonprofit four-year colleges and universities ranged from 90% to 93%.

Between 2008-09 and 2013-14, the increase in average institutional grant aid covered 95% of the \$4,000 increase in tuition and fees.

School Achieves Lower Profits

Corollary 2.

In the unobserved case, the school's expected profit Π^u is strictly lower than its expected profit Π^o in the observed case.

Idea

- ▶ In equilibrium, firms have correct belief and offer lower wages.
- ▶ This reduces the worker's willingness to pay for education.
- ▶ Cournot (unobserved case) vs Stackelberg (observed case).

School Charges Lower Tuition

Proposition 4.

$T^u(z) < T^o(z)$ in the common domain $(z^o(\underline{\theta}), z^o(\bar{\theta}))$.

Marginal Tuition

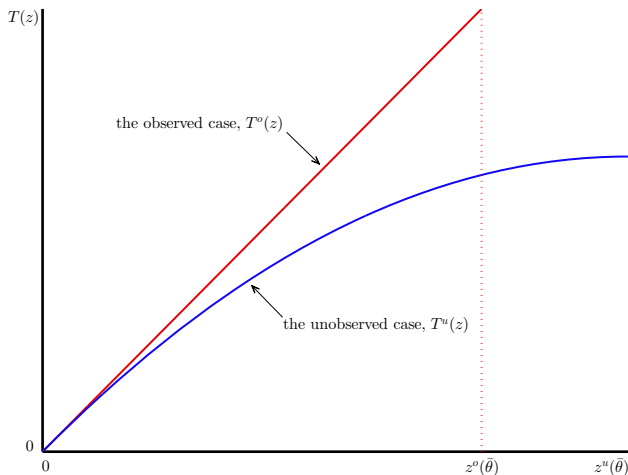
- ▶ In the observed case

$$T^{o'}(z) = Q_{\theta}(z, \theta^o(z)) \cdot \theta^{o'}(z) + \frac{1 - F(\theta^o(z))}{f(\theta^o(z))} [-C_{z\theta}(z, \theta^o(z))].$$

- ▶ In the unobserved case

$$T^{u'}(z) = \frac{1 - F(\theta^u(z))}{f(\theta^u(z))} [-C_{z\theta}(z, \theta^u(z))].$$

- ▶ School offers quantity discounts for higher education levels.



Assumptions: $Q(z, \theta) = \theta z + z$, $C(z, \theta) = z^2 + z - \theta z$, and $\theta \sim U[0, 1]$.

Worker Obtains Higher Utility

Proposition 5.

$U^u(\theta) \geq U^o(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality for $\theta > \underline{\theta}$.

Idea

- ▶ In equilibrium, the market belief over tuition is correct.
- ▶ Tuition is lower in the unobserved case, benefiting worker.

Worker Obtains Higher Utility

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Idea

- ▶ In equilibrium, the market belief over tuition is correct.
- ▶ Tuition is lower in the unobserved case, benefiting worker.

Implication

- ▶ Tuition transparency allows schools to commit to high tuition.
- ▶ Mandatory disclosure policies, such as U.S. Code § 1015a, may *unintentionally* raise education expenses and harm students.

Under-Education and Over-Education Coexist

Assumption 1.

The function

$$Q_{\theta}(z^{fb}(\theta), \theta) \cdot \theta^{fb'}(z^{fb}(\theta)) + \frac{1 - F(\theta)}{f(\theta)} C_{z\theta}(z^{fb}(\theta), \theta) \quad (*)$$

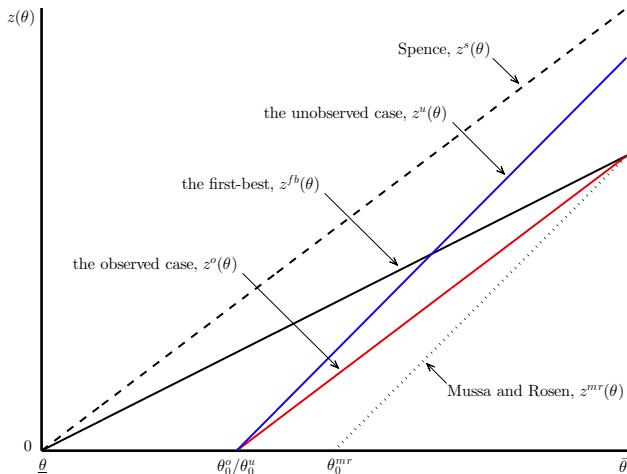
is single-crossing in θ .

Proposition 6.

There is a unique $\theta^w \in (\theta^, \bar{\theta})$ such that $z^u(\theta) < z^{fb}(\theta)$ on $[\underline{\theta}, \theta^w)$ and $z^u(\theta) > z^{fb}(\theta)$ on $(\theta^w, \bar{\theta}]$, where $\theta^* > \theta_0^u$ is the root of $(*)$.*

Proposition 7.

In the unobserved case, the worker chooses strictly less education than in Spence's signaling game, that is, $z^u(\theta) < z^s(\theta)$ on $[\underline{\theta}, \bar{\theta}]$.



Assumptions: $Q(z, \theta) = \theta z + z$, $C(z, \theta) = z^2 + z - \theta z$, and $\theta \sim U[0, 1]$.

Welfare and Signaling Intensity

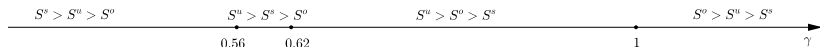
Parameterization

- ▶ $Q(z, \theta) = \gamma\theta z + z$, $\gamma > 0$; $C(z, \theta) = z^2 + z - \theta z$; $\theta \sim U[0, 1]$.
- ▶ $z^{fb}(\theta) = \frac{(\gamma+1)\theta}{2}$, $z^s(\theta) = \frac{(2\gamma+1)\theta}{2}$; Define signaling intensity:

$$\frac{z^s(\theta) - z^{fb}(\theta)}{z^{fb}(\theta)} = \frac{\gamma}{\gamma + 1}.$$

Thus, the greater γ is, the more intense signaling is.

- ▶ $z^o(\theta) = \frac{(\gamma+2)\theta-1}{2}$, $z^u(\theta) = (\gamma + 1)\theta - \frac{\gamma+1}{\gamma+2}$; Cutoff $\theta^w = \frac{2}{\gamma+2}$.
- ▶ The welfare comparison among the three cases is given by



- ▶ Competition among sellers is not necessarily socially beneficial.

Summary

We studied a signaling model in which

- ▶ Principal sets a price schedule for a good with intrinsic value.
- ▶ Agents choose how much to buy as a signal to the market.
- ▶ The market *may* or *may not* observe the price schedule.

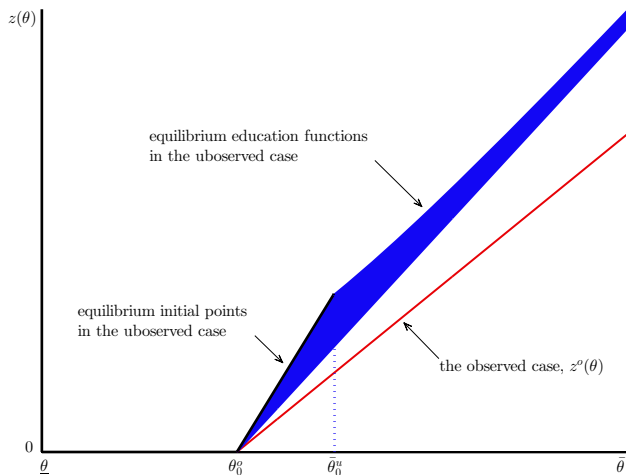
Main results

- ▶ In the observed case, agents buy less than the first-best.
- ▶ In the unobserved case, agents buy more than in the observed case; those of the highest types buy more than the first-best.

Next step

- ▶ *Competitive Nonlinear Pricing for Signals*, working paper.

Separating Equilibria of the Unobserved Case



Assumptions: $Q(z, \theta) = \theta z + z$, $C(z, \theta) = z^2 + z - \theta z$, and $\theta \sim U[0, 1]$.