Optimal Sequence for Teamwork

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Motivation

Peer information is critical to organization design

- Employees' job attitudes respond to observed efforts of peers.

Various workplace architectures in modern organizations

- Dense "open space" models, e.g., Bloomberg, Google, etc.
- Loose-fitting designs, e.g., Amazon's Virtual Locations, etc.
- Determines how information *can* flow internally among peers.

Team designer can often dictate the order of tasks

- Determines how information *will* flow internally among peers.

Key question: given a workplace architecture

- What is the optimal sequence of task execution for teamwork?

Overview

Model

- Moral hazard in team with network-based peer monitoring.
- Optimal effort-inducing mechanism: sequence and rewards.

Networks composed of stars

- Simple algorithm to derive the optimal mechanism.
- Balancing magnitude and coverage of incentive cost reduction.

Implications

- Transparency of agents' actions can reduce incentive costs.
- More important agents act later and earn higher rewards.
- Better connected agents involve in intermediate stages.

Literature

Theories on incentive design for teamwork

- Holmstrom (1982), Holmstrom & Milgrom (1991), Itoh (1991)
- McAfee & McMillan (1991), Segal (1999), Babaioff et al. (2012)

  Balmaceda (2016, 2018), Che and Yoo (2001), Segal (2003), Bernstein and Winter (2012), Winter (2004, 2006, 2010)

Empirics on the role of internal information

- Ichino & Maggi (2000), Teasley et al. (2002), Heywood & Jirjahnm (2004), Gould & Winter (2009), Mas & Moretti (2009)
- Carpenter et al. (2009), Steiger & Zultan (2014)

Model

Model

Players and actions

- A set I of n agents manage a project owned by a principal.
- Each agent chooses whether to exert effort or not: $a \in \{1, 0\}$.
- Exerting effort is costly $(c = 1)$ while shirking is costless.

Network

- Organizational structure is given by an undirected graph g.
- Agents i and j are directly linked if $(i, j) \in g$.

Technology

- The project succeeds with probability $p(W)$, $W = \{i | a_i = 1\}$.
- Monotonicity: if $T \subset S$, then $p(T) < p(S)$.

Mechanism

- Principal chooses a mechanism $\{\pi, v\}$ that consists of:

1. A sequence of execution (permutation) π.
2. A reward scheme v s.t. agent i receives $v_i \geq 0$ upon success.

Information

- Principal observes *only* whether the project succeeds or not.
- Agent i observes j's action (i sees j) iff $(i, j) \in g$ and $\pi_i > \pi_j$.
- Let $N_i \equiv \{j | (i, j) \in g, \pi_i > \pi_j\}$ be the agents whom i sees.
- Let $M_i \equiv \{j | j$ sees k_1 sees $\ldots k_r$ sees $i\}$ be the agents who could ultimately learn i's action if agents could communicate.

Principal's Problem

Agent's payoff

- A strategy of agent i is a mapping $s_i : 2^{N_i} \to \{0, 1\}$.
- Given a strategy profile s, agent i's expected utility equals

$$u_i(s) = p(W(s))v_i - s_i.$$

Effort-inducing (*EFI*) mechanism

- A mechanism is *EFI* if there exists a *PBE* s^* s.t. $W(s^*) = I$.
- **Principal's problem** is to design an *EFI* mechanism yielding minimal total payoffs to agents among all *EFI* mechanisms.
- Denote the optimal EFI mechanism $\{\pi^*, v^*(\pi^*)\}$.

More on Technology

Definition: Importance

- Agent i is more important than j if for any S s.t. $i, j \in S$,

$$p(S \backslash \{i\}) \leq p(S \backslash \{j\}).$$

Definition: Complementarity (increasing return-to-scale)

- p is complementary if for any S, T s.t. $T \subset S$ and for $i \notin S$,

$$p(S \cup \{i\}) - p(S) > p(T \cup \{i\}) - p(T).$$

Definition: Substitutability (decreasing return-to-scale)

- p is substitutable if for any S, T s.t. $T \subset S$ and for $i \notin S$,

$$p(S \cup \{i\}) - p(S) \leq p(T \cup \{i\}) - p(T).$$

Preliminary Analysis

Optimal Reward Scheme under Complementarity

Proposition 1.

Suppose that p is complementary. For any fixed permutation π, the optimal reward scheme $v^*(\pi)$ exists and pays agent i

$$v_i^* = [p(I) - p(I \backslash (\{i\} \cup M_i))]^{-1}.$$

Proof sketch

- s^* is a PBE: $s_i^* = 1$ iff $a_j = 1$ for all $j \in N_i$ or N_i is empty.
- Suppose a PBE comprises a $v_i < v_i^*$, then agent i will shirk.

Idea

- Agents face *implicit threats of shirking* acting sequentially.
- A more transparent action results in a lower incentive cost.

Optimal Reward Scheme under Substitutability

Proposition 2.

Suppose that p is substitutable. For any fixed permutation π, the optimal reward scheme $v^*(\pi)$ is identical and pays agent i

$$v_i^* = [p(I) - p(I \backslash \{i\})]^{-1}.$$

Proof sketch

- s^* is a PBE: $s_i^* \equiv 1$.
- Suppose a PBE comprises a $v_i < v_i^*$, then agent i will shirk.

Idea

- Agents substitute own efforts for observed shirking actions.
- Peer information thus does not reduce incentive costs.

Fully Connected Network

Fully Connected Network

Definition: Fully connected network

- All agents are interconnected.

Assumption 1.

From now on, assume that p is a complementary technology.

Lemma 1.

For any two agents i and j s.t. $(i, j) \in g$, we have $\pi_i^* \neq \pi_j^*$.

Idea

- Simultaneous moves reduce the transparency of actions.

Implication

- In fully connected networks, agents act in the order $1, \ldots, n$.

Optimal Sequence

Identity permutation

- Relabel agents s.t. agent i is less important than $i+1$.

Proposition 3.

(i) *For any two agents* i *and* j *s.t.* $(i,j) \in g$, i *is more important than* j *and* $\{k|(i,k) \in g\} \backslash \{j\} = \{k|(j,k) \in g\} \backslash \{i\}$, *if in the optimal sequence* π^*, $|\pi_i^* - \pi_j^*| = 1$, *then* $\pi_i^* = \pi_j^* + 1$.

(ii) *Thus, if* g *is a fully connected network and the agents are increasingly important, then* π^* *is the identity permutation.*

Idea

- Shirking triggers all the successors to shirk.
- More important successors mean a greater implicit threat.

Differential Rewards

Corollary 1.

Suppose that g is a fully connected network, then in the optimal reward scheme $v^*(\pi^*)$: v_i^* is increasing and strictly convex in i.

Idea

- Agent in a later stage can free ride on more preceding efforts.
- Free riding is more detrimental as more agents exerted efforts.
- Thus, agents are increasingly more difficult to be incentivized.

Implication

- Differential rewards maybe inevitable even for identical agents.

Optimality of Fully Connected Network

Corollary 2.

A fully connected network yields minimal total payoffs to the agents, and thus maximal payoff to the principal.

Idea

- Fully connected network yields the richest transparency.

Implication

- “Open space” improves peer monitoring and reduces shirking.

Star Network

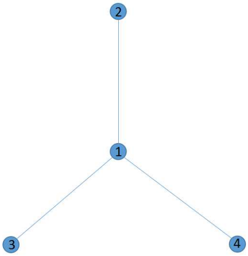
Star Network

Definition: Star network

- There is a unique node i s.t. every link in g involves i;
- Agent i is called the *center*, and the others the *peripheries*.

Star networks are common in organizations

- E.g.: scientific labs, large-scale construction contractors.



Center's Successors are More Important Peripheries

Lemma 2.
For any two agents i and j s.t. $\{k|(i,k) \in g\} = \{k|(j,k) \in g\} \neq \emptyset$ and i is more important than j, if in the optimal sequence π^*, there exists some $k' \in \{k|(i,k) \in g\}$ s.t. $\pi_i^* \wedge \pi_j^* < \pi_{k'}^* < \pi_i^* \vee \pi_j^*$, then $\pi_i^* > \pi_j^*$.

Implication

- Center's successors are more important than his predecessors.

A sufficient statistic for optimum

- Let m be the number of center's successor(s), $0 \leq m \leq n-1$.

Total Payoffs to Agents

Identity permutation

- Relabel peripheries with rising importance from 1 to $n-1$.
- Let center be the n-th (not necessarily the most important).

According to Proposition 1,

- Total payoffs under an optimal reward scheme is given by

$$v^*(m) = \underbrace{\sum_{i=1}^{n-1-m} \frac{1}{p(I) - p(\{j|j < n-m\} \backslash \{i\})}}_{\text{payoffs to the predecessors}} + \underbrace{\frac{1}{p(I) - p(\{j|j < n-m\})}}_{\text{payoff to the center}} + \underbrace{\sum_{n-m}^{n-1} \frac{1}{p(I) - p(I \backslash \{i\})}}_{\text{payoffs to the successors}}.$$

Impacts of Increasing Center's Successors

Marginal benefit of raising m

- Reduces the total payoffs to center and his predecessors by

$$MB(m) := \sum_{i=1}^{n-2-m} \left[\frac{1}{p(I) - p(\{j|j < n-m\} \backslash \{i\})} - \frac{1}{p(I) - p(\{j|j < n-m-1\} \backslash \{i\})} \right] + \frac{1}{p(I) - p(\{j|j < n-m\})}$$

Marginal cost of raising m

- Increases the total payoffs to center's successors by

$$MC(m) := \frac{1}{p(I) - p(I \backslash \{n-m-1\})}$$

Optimal Sequence

Lemma 3.

$MB(m)$ is decreasing whereas $MC(m)$ is non-decreasing in m.

Implication

- There is a simple algorithm to find the optimal sequence.

Proposition 4.

The optimal sequence π^* satisfies that

(i) *Center has m^* successor(s) with $0 \leq m^* \leq n-2$ and each of them is more important than all the center's predecessors.*

(ii) *Furthermore, if $\left[p(I) - p(I \backslash \{n-1\})\right] < \delta \left[p(I) - p(I \backslash \{n\})\right]$ for a sufficiently small $\delta > 0$, then $m^* = 0$.*

Comparative Statics: Importance of Individual Task

Parametric setting (Winter 2006)

- Let w be the number of agents who exert effort, $w \leq n$.
- The probability of success is $p(w) = \alpha^{n-w}$, $\alpha \in (0, 1)$.
- Lower α means the failure of individual task is more crucial.

Corollary 3.

In the optimal sequence π^*, the number of the center's successors $m^*(\alpha)$ is non-decreasing in α for $\alpha \in (0, 1)$.

Idea

- If each task is important, agents are more willing to work.
- Implicit threat of shirking is not crucial in providing incentive.
- Thus, marginal benefit of raising m is relatively low.

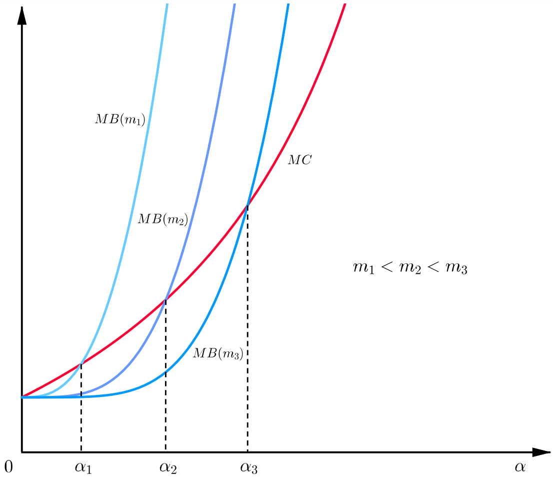


Figure 1: Marginal Benefit and Marginal Cost as a Function of Importance

Core-Periphery Network

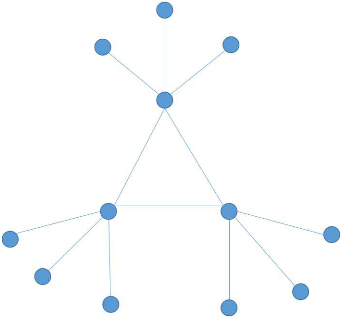
Core-Periphery Network

Definition: Core-Periphery network

- Multiple stars with the centers interconnected.

Core-Periphery networks are common in organizations

- E.g.: development of new drugs, universities.



Nature of Production Process

Definition: Vertical project

- The relative order between different stars is fixed while that of the agents within each star is determined by the principal.
- Multi-phase projects with vertical collaboration.

Definition: Horizontal project

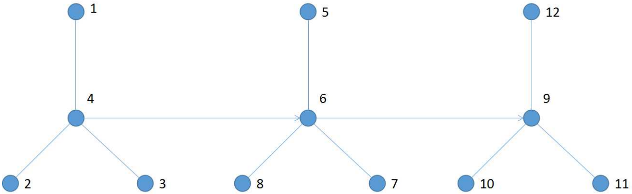
- The relative order between different stars is also adjustable.
- Projects with horizontal collaboration of multi-departments.

Assumption 2.

All the agents are equally important to the project.

Vertical Projects

- Let r be the number of stars in the network.
- Let s_i be the number of agents in star i, $\sum_{i=1}^{r} s_i = n$.
- Principal chooses the sequence within each star separately.
- Yields the same transitive closure as a connected-star network:



Impacts of Increasing Center's Successors

- Let m_i be the number of center i's successors within star i.

Marginal benefit of raising m_i

- Has now an inter-star effect: reduces the payoff to the centers and his predecessors within own star and all the previous stars.

Marginal cost of raising m_i

- Increases the total payoffs to center's successors by

$$MC \equiv \frac{1}{p(n) - p(n-1)}$$

Inner-Star Sequence

Lemma 4.

The optimal sequence π^* satisfies that

(i) *If there is a star i s.t. $s_i - 1 - m_i^* > 0$, then $m_j^* = 0$, $\forall j < i$.*

(ii) *If there is a star i s.t. $m_i^* > 0$, then $m_j^* = s_j - 1$, $\forall j > i$.*

Idea

- Marginal benefit of raising m_i is higher for smaller i.

Implication

- Each star can have at most three possible internal sequences:
  1. *Type I*: the center acts the last within own star.
  2. *Type II*: the center acts in an interior stage within own star.
  3. *Type III*: then center acts the first within own star.
- Lemma 4 means that there is at most one type II star.

Optimal Sequence for Vertical Projects

Proposition 5.

The optimal sequence π^* can be obtained through the procedure:

1. *Make each star a type I star.*
2. *From the last star to the first, assign the peripheries one by one to the center's successors, until $MB(m_i^*) \leq MC$ for some star i in which m_i^* peripheries act after the center.*

No center acts the first or the last in the entire sequence.

Horizontal Projects

- Lemma 1 implies that all stars should act sequentially.
- It remains to determine the relative order between stars.

Proposition 6.

The optimal sequence π^* satisfies: for any $1 \leq i \leq r-1$,

(i) *If both stars i and $i+1$ are type I stars, then $s_i \geq s_{i+1}$.*

(ii) *If both stars i and $i+1$ are type III stars, then $s_i \leq s_{i+1}$.*

Implication

- If types I and III coexist, then the permutation is "V-shaped".
- Proposition 6 serves to exclude most suboptimal ones.

Conclusion

Optimal effort-inducing mechanism for teamwork

- Network-based internal information.
- Simple algorithm to derive the optimal sequence.

Individual's importance and connectivity

- More important agents involve in later stages.
- Better connected agents involve in intermediate stages.

Future work

- Incentive design with more general networks.
- Incentive design with new channels of internal information.