Network-Based Peer Monitoring Design*

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Abstract

We study a team incentive design problem where multiple agents are located on a network and work on a joint project. The principal seeks the least costly mechanism to incentivize full effort, by choosing the work assignment sequence and the rewards to the agents upon success. Whereas the agents' actions are hidden to the principal, they may be observed among the agents given the internal information that is determined by the network and the sequence. Under effort complementarity, the transparency of the agents' actions can reduce their incentive costs, but exhibits diminishing marginal effectiveness. This gives rise to the desire for balancing internal transparency when it is infeasible to uniformly enhance transparency. For several typical network topologies, we derive explicit properties of an optimal assignment sequence, and propose two new measures, total cost and stability, for the principal to rank these networks.

Keywords: Peer information, Task assignment, Network, Incentive design

JEL Classification: D82, L14, L23

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1 Introduction

In many organizations, incentivizing efficient teamwork depends critically on exploiting *internal information*. That is, while an agent's effort is unobserved externally by the principal, it can be peer-monitored internally among agents, in the sense that actions by early movers in a team are observed by some late movers. The principal then leverages on such observation structure to reduce incentive costs. Meanwhile, a prominent feature of modern organizations is flexible task assignment: Unlike in conventional waterfall teams, the principal can largely determine the order of completing individual tasks, e.g., in film production that involves independent crews in different locations, or for an NSF-funded R&D project that requires joint effort by various laboratories or universities. It therefore stands out as a challenging and important problem to design the optimal task assignment, which essentially generates a peer-monitoring structure. Referring to Holmstrom (1982)'s classic remark, "...monitoring technologies were exogenously given. In reality, they are not. The question is what determines the choice of monitors; and how should output be shared so as to provide all members of the organization (including monitors) with the best incentives to perform?"

As pointed out by relevant literature (e.g., Maskin et al. (2000)), tackling this problem is closely related to understanding the influence of organizational form, as different organizational forms will give rise to different information about performance. In this paper, we adopt a general, *network-based* perspective on organizational forms, and study a principal who designs the peer monitoring structure in this context. The principal faces an exogenous network topology governing available internal information among agents, and endogenously chooses the sequence of task assignment and the associated rewards. We contribute to the literature by identifying two key leverages for the principal to exploit internal information and minimize incentive costs, as well as how they are explicitly reflected by the properties of optimal sequences under various types of network topologies. We also propose a number of analytical tools, such as algorithms for identifying an optimal sequence and measures for ranking network topologies by the principal's preference.

In the model, a number of teams collaborate on a risky project. Every team consists of multiple agents, with one being the manager and the others subordinates, and each agent is responsible for an individual task. An agent can increase the chance of the project's success by exerting costly effort, while the exact marginal effect of his effort depends on other team members' inputs. Throughout the paper, we mainly focus on complementary technologies, i.e., the probability of success is supermodular in agents' efforts. Before performing his task, an agent can potentially observe the efforts chosen by earlier movers. The observability of effort, or peer monitoring structure, is governed by an exogenous and undirected network, which dictates the connections between agents within and across teams. If two agents i and j are linked, then j can observe whether i exerted effort when j moves after i, and vice versa.

The principal cannot observe the effort choice of any agent but is aware of the network topology, and aims at inducing effort from each agent at the lowest possible costs. Our main departure from the literature on incentive design in teams is the set of available mechanisms that the principal can choose. Specifically, the principal determines (1) the work sequence, i.e., the sequence of moves by the agents, and (2) the reward to each agent upon success of the project. That is, a feasible mechanism consists of both pecuniary and non-pecuniary incentive instruments. To the best of our knowledge, this paper is the first to investigate the optimal work sequence in a team incentive design problem.

Drawing on relevant literature such as Winter (2010), we have a ready characterization of the optimal reward scheme given a fixed sequence of moves. Hence, our analysis focuses on the novel element of designing an optimal sequence, which is both a sequence of moves and a sequence of peer monitoring, given the network topology. We start by deriving the optimal sequence for a single team, whose network topology takes the form of either a star or a clique. The results demonstrate two fundamental economic forces that the principal may leverage for reducing incentive costs. First and intuitively, the principal always benefits if she can improve the internal transparency of each agent's action. This implies that the total incentive cost under *any* network is bounded below by that under a clique network, in a sequence where the agents move one after another and each observes their immediate predecessor. The second and more novel force is diminishing marginal benefit from internal transparency, implying that the principal seeks to properly balance internal transparency when uniform improvement of transparency is impossible. For example, under a star network, a trade-off typically arises between the rewards offered to the manager's predecessors (the monitored) and those to the successors (the monitors), when the principal compares two different sequences. Nevertheless, we show that the marginal total incentive cost is monotone in the number of the manager's successors, and an optimal sequence can thus be identified by a straightforward algorithm. This sequence always assigns less subordinates as monitors than as monitored agents, which stems from balancing internal transparency.

We then focus our main analysis on collaborating teams of possibly heterogeneous sizes. We examine two classes of networks based on how the team managers are connected. The first class includes flat organizations, where the managers form a clique, and can be categorized as non-divisional or divisional according to whether managers are linked to subordinates in other teams. For each category, we further divide the networks by the topology within teach team, which is either a clique or a star. The second class includes hierarchical organizations, where the managers are connected through a tree network. Similarly, we divide the class into hierarchical cliques and hierarchical stars, according to the topology within each team. The following table summarizes the types of networks examined in this paper.

<table>
<tr><td rowspan="2">Within\Across teams</td><td colspan="2">Flat: managers form a clique</td><td rowspan="2">Hierarchical: managers form a tree</td></tr>
<tr><td>Non-divisional: manager linked to other teams' subordinates</td><td>Divisional: manager not linked to other teams' subordinates</td></tr>
<tr><td>Clique</td><td>Dense core-periphery</td><td>Connected cliques</td><td>Hierarchical cliques</td></tr>
<tr><td>Star</td><td>Core-periphery</td><td>Connected stars</td><td>Hierarchical stars</td></tr>
</table>

Table 1: Categories of networks for multi-team organizations.

While the type of network varies, the principal consistently relies on the aforementioned leverages—uniformly improving internal transparency, or balancing internal transparency based on diminishing marginal benefit if necessary—when designing an optimal sequence. However, the network topology largely determines the manifestation of either leverage, i.e., the same economic force may induce entirely distinct properties for different networks. In non-divisional networks, improving internal transparency requires that no two managers are assigned adjacent to one another in an optimal sequence, but there is a group of subordinates moving between each two managers. Diminishing marginal benefit further implies that the group size of these subordinates must *decrease* over time. In divisional networks, improving internal transparency serves as the dominant force in shaping an optimal sequence, but predicts a *non-monotone* pattern in team size over time. In the special case of homogeneous team sizes, one can resort to diminishing marginal benefit and identify an optimal sequence via a simple algorithm. Finally, the characterization of an optimal sequence in hierarchical networks resembles that for a star: A number of managers move before the top manager (root of the tree) with all or most subordinates as their predecessors, while the other managers move after the top manager with all or most subordinates as their successors. The former group is typically larger than, or not much smaller than, the latter group.

In response to Holmstrom (1982)'s remark, the assignment of roles between monitors and the monitored is now endogenous and again depends crucially on the network topology. In networks with rich global (intra-team) connections, e.g., non-divisional networks, most agents do not serve as either role, but essentially become "intermediaries" who monitor all predecessors and are monitored by all successors. By contrast, in divisional or hierarchical networks, the managers are intermediaries while every subordinate is endogenously made a monitor (i.e., moving after their manager) or a monitored agent (i.e., moving before their manager). As a uniform property for such networks, we find that monitors only take up an infinitesimal proportion of agents as the number of agents grows to infinity. This observation is the product of two forces. On the one hand, each monitor's action remains unobserved or is only observed by a limited number of peers, so incentivizing a monitor's effort is relatively expensive. On the other hand, diminishing marginal benefit of transparency implies that too many monitors would prove inefficient for lowering the incentive costs of monitored agents after all.

Based on the characterization of optimal sequences, we further develop two measures by which the principal may compare different network structures. The first is the measure of total cost, where we identify the minimal set of essential links for a sequence by transitive reduction, and impose a maintenance cost for every such link. The principal then chooses a sequence to minimize the sum of maintenance costs and incentive costs. The second is the measure of stability, where we calculate the maximum fraction of links in a network that can be severed, conditional on preserving the transitive reduction for minimizing incentive costs. Although the two measures differ significantly by nature, we find very consistent results for comparing the above types of networks. With either measure, the principal always prefers a network to another if the former offers higher internal transparency of each agent's action. Therefore, the network structure most favorable to the principal is the dense core-periphery network, and the least favorable network is the hierarchical-stars network. A more interesting comparison is between a core-periphery network and a connected-cliques/hierarchical-cliques network, noting that the former represents rich global connections while the latter rich local connections. We find that, with homogeneous team sizes and by either measure, the principal prefers the former network when team size is fixed and team number extends to infinity, but the latter network when team number is fixed and team size extends to infinity.

The rest of the paper is organized as follows. Section 1.1 below discusses related literature. Section 2 describes the model. Sections 3 and 4 provide preliminary analysis for the optimal reward, as well as characterization of optimal sequences in a single team. Section 5 presents the main results for optimal sequences in collaborating teams. Section 6 introduces measures for network comparison and the related results. Section 7 concludes our paper. All proofs are in the Appendix.

1.1 Related literature

The theoretical literature on incentive design for teamwork is extensive and growing. The trade-off an agent faces between working and shirking originated from the classical literature on moral hazard in teams (Alchian and Demsetz, 1972; Holmstrom, 1982; Holmstrom and Milgrom, 1991; Itoh, 1991). Subsequent studies developed this literature to static contracting on teamwork with a number of variations, such as externalities (McAfee and McMillan, 1991; Segal, 1999; Babaioff et al., 2012), specialization versus multitasking (Balmaceda, 2016), loss-averse agents (Balmaceda, 2018), network-based production spillover (Sun and Zhao, 2021), and network-based equity compensation (Dasaratha et al., 2023). Our main contribution to this literature is to consider the endogenization of internal information among the agents in the presence of moral hazard.

Several recent papers have investigated how including or altering the scheme of information sharing among agents affects incentive design. Zhou (2016) shows that the welfare-optimal organization of team members is a chain when the first mover observes the state of nature and the later movers observe their immediate predecessor's effort. Our analysis produces a similar result when the exogenous network of internal information is complete. Gershkov et al. (2016) study the efficient contract design given that some team member may share information about a payoff-relevant state, and they show that efficiency can be achieved if contracts take into account a contest ranking across agents. Au and Chen (2021) characterize the optimal long-term contract in teams of two members, with efforts observable between the paired agents. In Camboni and Porcellacchia (2021), the principal observes noisy signals about efforts, and may condition the contract offered to each team member on both her individual signal and the whole project's outcome. The optimal incentive scheme features a partition between insulated and non-insulated agents ranked by signal precision.

A comprehensive study on the role of internal information in effort-based teamwork, with an exogenous sequence of task assignment, has been provided by Winter (2004, 2006, 2010). This is the main strand of literature we follow on building the theoretical framework. Our results indicate that some peer information architectures are more likely to emerge than others, once the sequence becomes the principal's choice. Halac et al. (2021) also investigate the incentive design problem in teamwork in the presence of moral hazard, but the principal leverages uncertainty of ranking among agents instead of internal information. Consequently, they show that discrimination is suboptimal in contrast to Winter (2004). Gershkov and Winter (2015) study the optimal incentive design with fixed work sequence, random peer monitoring and the principal's choice of costly individual monitoring, and they show that peer monitoring substitutes for the principal's monitoring when the production technology is complementary. The broad idea of the interplay between early and late movers in teams, and how a designer can exploit such structure for efficiency or cost saving, have also been studied from other perspectives including information-based leadership (Hermalin, 1998; Zhou and Chen, 2015; Zhou, 2016), tournaments in team production (Gershkov et al., 2009, 2016) and various forms of payoff externalities (Che and Yoo, 2001; Segal, 2003; Bernstein and Winter, 2012). In addition, Xiang (2020) considers a simultaneous-move team moral hazard problem where an agent may bribe their monitors, and shows that an optimal monitoring structure must have a core-periphery topology.

The importance of internal information in incentive design has also been noted by data. Empirical evidence suggests that workers' productivity and willingness to work respond positively to observed efforts of peers (Ichino and Maggi, 2000; Heywood and Jirjahn, 2004; Gould and Winter, 2009; Mas and Moretti, 2009). Experimental studies on behavior in team production have also indicated that an agent's contribution in teamwork is highly responsive to internal information (Carpenter et al., 2009; Steiger and Zultan, 2014) and that unequal rewards tend to facilitate coordination and improve efficiency (Goerg et al., 2010).

2 Model

Players and actions. A principal (she) owns a project that is collectively managed by a set I of n agents. The agents are grouped into a set T of teams. Each team $t \in T$ consists of a *manager*, denoted l_t, and $f_t \geq 2$ *subordinates*; each agent's role will be specified later. Each agent i (he) is responsible for an individual task, and chooses $a_i \in A_i \equiv \{0, 1\}$, with $a_i = 1$ if he exerts effort, and $a_i = 0$ if he shirks. The cost of effort is 1 across all the agents, whereas shirking is costless. Henceforth, we use the terms *work* and *exert effort* interchangeably.

Technology. The organization's technology is a mapping from a profile of effort levels to a probability of the project's success. Given a subset $I' \subseteq I$ of working agents, the probability of success is $p(I')$. We assume that p is increasing such that if $I'' \subset I'$, then $p(I'') < p(I')$. Moreover, we assume that p satisfies *complementarity* in the sense that for any two subsets I' and I'' with $I'' \subset I'$ and any agent $i \notin I'$, $p(I' \cup \{i\}) - p(I') > p(I'' \cup \{i\}) - p(I'')$; that is, i's effort is more effective if the set of other working agents enlarges.[1] We also distinguish between different agents' *importances* to the project. We say that agent i is (weakly) more

[1] Conversely, p satisfies *substitutability* if the inequality changes direction. It follows from Winter (2010, Proposition 2) that under substitutability, each agent's optimal reward and thus the optimal mechanism will be invariant with the internal information (see below for details). Thus, we do not consider substitutability.

important than j if for any subset I' with $i, j \in I'$, we have $p(I' \backslash \{i\}) \leq p(I' \backslash \{j\})$; that is, i's shirking is more detrimental than j's to the chance of success. We assume that the set I is totally ordered in terms of agent importance.

Network. The agents are connected by a deterministic and undirected network g. We write $ij \in g$ to indicate that agents i and j are directly linked, and say that i and j are *neighbors*. In particular, $ii \notin g$ for any agent i, and the assumption that g is undirected implies that $ij \in g \Leftrightarrow ji \in g$. In applications of our framework, network g could capture the workplace architecture, the authority structure, geographical locations, informal social networks and so forth. We assume that the structure of g is common knowledge.

Mechanism. Before the agents perform their tasks, the principal designs a work sequence, or simply a *sequence*, π, such that agent i is the π_i-th player to move, with $\pi_i \in \{1, \ldots, n\}$. The sequence π can be interpreted from dual perspectives. In agile projects such as software development and market campaigns, the work sequence has become remarkably more flexible, allowing for adjustments. In this context, π can be interpreted quite literally. Conversely, in projects with a less flexible work sequence but versatile agents competent in diverse tasks, π can be seen as the assignment of tasks among these agents. In addition to the sequence, the principal designs a reward scheme $v = (v_1, \ldots, v_n)$, such that agent i receives $v_i \geq 0$ if the project turns out to be successful, and receives zero reward otherwise. The principal cannot monitor the agents' efforts, but simply knows whether the project is successful after all the tasks have been performed. In summary, a *mechanism* $\{\pi, v\}$ consists of a sequence π and a reward scheme v. We assume that the principal can commit to the mechanism.

Internal information. The agents' *internal information* about their peers' effort levels is jointly determined by the network g and the sequence π. Specifically, agent i observes agent j's action, or simply i *sees* j, before i moves if and only if i and j are neighbors and i moves after j.[2] That is, $ij \in g$ means that i *can* see j based on the network, and i *will* see j when he moves after j. As such, the sequence π determines a directed network based on g, with each arc indicating who sees whom. We call such a directed network a *monitoring network*. For each π, we define $N_i(\pi) := \{j | ij \in g, \pi_i > \pi_j\}$, N_i for short, to be the set of agents whom agent i sees in the resulting monitoring network.

Principal's problem. Consider the game that is defined by the set of agents I, the agents' action space $\{A_i\}_{i \in I}$, the network g and a mechanism $\{\pi, v\}$. A (pure) strategy of agent i is

[2] If i and j move simultaneously, then neither of them can see the other.

a mapping $\sigma_i : 2^{N_i} \to \{0, 1\}$, which specifies i's action as a function of his information about other agents' efforts. Given a strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$, agent i's payoff is given by

$$U_i(\sigma) \coloneqq p(W(\sigma))v_i - \mathbb{1}(\sigma_i = 1),$$

where $W(\sigma)$ is the set of working agents given σ, and $\mathbb{1}(\cdot)$ is the indicator function.

A mechanism $\{\pi, v\}$ is *effort-inducing (EFI)* with respect to the network if there exists a perfect Bayesian equilibrium (PBE) in the resulting game, in which every agent exerts effort. The principal's problem is to design an EFI mechanism that yields minimal total rewards to the agents among all EFI mechanisms, which is called an optimal mechanism. In particular, given a sequence π, a reward scheme $v^*(\pi)$ is optimal if $\{\pi, v^*(\pi)\}$ is an optimal mechanism. The principal's objective is meaningful when the project's value is sufficiently high and the agents' efforts are efficient to raise the probability of success. Alternatively, one can consider the mechanisms that maximize the principal's expected net revenue, but we refrain from this approach, as it does not provide new insights while complicates the analysis remarkably.

3 Preliminary analysis

3.1 Optimal reward scheme and action transparency

As a first step, we characterize the optimal reward scheme for a fixed sequence π, applying Winter (2010)'s main result.[3] Define $M_i(\pi)$, M_i for short, to be the set of agents such that for every $j \in M_i$, there exists a sequence $\{k_r\}$ such that j sees k_1 sees k_2 sees $\ldots k_r$ sees i. We call the agents in M_i those who can *learn* i's action based on the idea that everyone in M_i would be informed of i's action if an agent could communicate with those who see him. The lemma below characterizes the optimal reward scheme $v^*(\pi)$ for an arbitrary π.

Lemma 1. *For any fixed sequence π, the optimal reward scheme $v^*(\pi)$ pays agent i a reward equal to $v_i^*(\pi) = [p(I) - p(I\backslash(\{i\} \cup M_i))]^{-1}$.*

Intuitively, when agents move sequentially, they are facing an *implicit threat of shirking.* Specifically, the exposure of a low effort might induce an agent who observes this action to shirk and consequently triggers a *domino effect* of shirking, making success less likely. This implicit threat thus reduces the agent's incentive cost. Under a complementary technology

[3] Winter (2010) characterizes the optimal reward scheme for a fixed internal information structure under complementarity (see Winter (2010, Proposition 4)).

and the optimal reward scheme, it is indeed sequentially rational for an agent to shirk once he sees someone shirking, rendering the implicit threat credible.

Lemma 1 indicates that if agent i's action becomes more transparent in the sense that if the set M_i enlarges, then i should be paid less, because he is more willing to work under a larger implicit threat. That is, with effort complementarity, action transparency can reduce an agent's incentive cost. Furthermore, Lemma 1 implies that the marginal effect of action transparency is diminishing as the action becomes more transparent. To see this, suppose a subset I' of agents who could not learn agent i's action now learn his action, then the change in v_i^* equals $[p(I) - p(I\backslash(\{i\} \cup M_i))]^{-1} - [p(I) - p(I\backslash(\{i\} \cup M_i \cup I'))]^{-1}$, or equivalently,

$$\frac{p(I\backslash(\{i\} \cup M_i)) - p(I\backslash(\{i\} \cup M_i \cup I'))}{[p(I) - p(I\backslash(\{i\} \cup M_i))][p(I) - p(I\backslash(\{i\} \cup M_i \cup I'))]}. \tag{1}$$

Note that as the set M_i enlarges, the numerator decreases due to complementarity, while the denominator increases due to monotonicity. Thus, the ratio decreases. That is, the marginal reduction in i's reward diminishes as his action becomes more transparent. Intuitively, under complementarity, a low effort is less detrimental to success when there are more agents who choose to shirk. This lowers the marginal benefit of action transparency. To summarize,

Corollary 1. *For any agent i, v_i^* is decreasing in M_i. Furthermore, suppose in sequences π and π', the set of agents who can learn i's action is given by M_i and M_i', respectively, where $M_i' = M_i \cup I'$ for some $I' \neq \emptyset$ with $I' \cap M_i = \emptyset$, then $|v_i^*(\pi') - v_i^*(\pi)|$ is decreasing in M_i.*

Corollary 1 indicates that while action transparency can reduce an agent's incentive cost, the marginal benefit of transparency diminishes when the action becomes more transparent. Put differently, the benefit function of action transparency is increasing but strictly concave. This novel insight will play a central role in achieving our main results. Since I is finite, the existence of an optimal mechanism is guaranteed by Lemma 1; thus, it remains to find such a mechanism by characterizing the optimal sequence. Let $V^*(\pi) := \sum v_i^*(\pi)$ denote the total rewards to the agents given the sequence π.

3.2 Properties of the optimal sequence

This section presents two general results that hold for any network g. First, we show that if two agents are neighbors, they cannot move simultaneously in the optimal sequence π^*.

Lemma 2. *For any two agents i and j, if $ij \in g$, then $\pi_i^* \neq \pi_j^*$.*

Intuitively, letting two neighbors move simultaneously reduces the transparency of their actions, thereby weakening the implicit threat of shirking and thus increasing incentive costs. In short, having two agents moving simultaneously is a waste of transparency.

Second, we show that if two agents share the same set of neighbors other than themselves and either one can learn the other's action in the optimal monitoring network, then the more important agent moves later. Formally, we have the following lemma.

Lemma 3. *For any two agents i and j such that $\{k|ik \in g, k \neq j\} = \{k|jk \in g, k \neq i\}$ and i is more important than j, if in π^* either $i \in M_j^*$ or $j \in M_i^*$, then $\pi_i^* > \pi_j^*$.*

Intuitively, if the more important agent i moves before the less important agent j, then it is profitable to switch their orders. Since i and j share the same set of neighbors other than themselves, swapping i and j would only affect the rewards of i, j and every agent k such that $j \in M_k$ and $i \notin M_k$, i.e., the agents whose actions can be learned by j but not i. In particular, the principal can reduce k's reward by replacing j's position with i, because if k shirks he will then trigger a more important agent to shirk, leading to a greater implicit threat. Analogously, i's new reward will be lower than j's old reward, while j's new reward equals i's old reward. Therefore, the principal can reduce the total rewards by assigning the more important agent to the later stage.

4 Single team

In this section, we consider the case in which there is only one team (i.e., T is a singleton). We fully characterize the optimal mechanism for two canonical network structures that are common in real-world organizations: *clique*, where the agents are fully connected, and *star*, where the manager is the center of the star, and the subordinates are the peripheral agents. This analysis serves two purposes. First, cliques and stars will function as the fundamental components for the subsequent analysis of multi-team organizations. Second, by examining these simple networks, we can clarify how the structure of exogenous network affects that of the optimal monitoring network, and how an agent's position in the network, as well as his importance to the project, jointly determine his role in the monitoring network.

4.1 Clique

Assume that the agents are fully connected. For example, many modern organizations have embraced open-space environments, or "war rooms". Such workplace architectures notably promote the transparency of employers' actions and the opportunities for peer monitoring. Therefore, such an environment can be considered a clique network. Lemma 2 implies that in the optimal sequence, the agents move sequentially in the order $1, 2, \ldots, n$. Thus, agents in later stages effectively serve as the monitors of the team, and will punish shirking actions by shirking as well. Moreover, Lemma 3 implies that the agents move in ascending order of importance; thus, the monitors are relatively more important. For ease of exposition, relabel the agents such that agent i is less important than $i+1$, $i \leq n-1$. The next proposition characterizes the optimal sequence in a clique network.

Proposition 1. *If g is a clique network and the agents are increasingly important, then the optimal sequence π^* is the identity permutation.*

Proposition 1 echos the main result of Winter (2006) where the agents must perform the tasks sequentially and can observe all preceding actions, such that in the optimal sequence the agents move in ascending order of importance. In our setting, within a clique network, such a sequence is not only feasible but also optimal. Thus, Winter (2006)'s main result can be regarded as a special case of ours. Moreover, the corollary below indicates that the total rewards to the agents are the least in a clique network among all network structures, since a clique network can generate the most transparent monitoring network.

Corollary 2. *A clique network generates minimal total payoffs to the agents, thereby maximizing the payoff to the principal.*

4.2 Star

Assume that the manager (center) is connected to all the subordinates (peripheral agents), each of whom is only connected to the manager. For example, in a scientific lab, the project leader typically plays a central role in the team, while each fellow researcher focuses on his or her individual task and reports progress exclusively to the leader. Such a team thus exhibits a star network structure. Other examples of star network might include a general contractor and subcontractors, a book editor and chapter contributors, and so forth.

To find the optimal sequence for a star network, note that it suffices to characterize the set of the center's successors, with the possibility of an empty set. For ease of exposition, we relabel the peripheral agents by importance from 1 to $n-1$, with a higher index referring to a more important agent. Provided there is no confusion, let the center be the n-th agent who is not necessarily the most important agent. Note that every peripheral agent has the same unique neighbor, i.e., the center. Then, by Lemma 3, we have the following lemma.

Lemma 4. *If in π^* the center has both a nonempty set of predecessors and a nonempty set of successors, then the successors are uniformly more important than the predecessors.*

The intuition of Lemma 4 has been suggested already; that is, if more important agents move in later stages, then a low effort will trigger agents with higher importance to shirk and is thus more detrimental to success, thereby allowing the principal to reduce incentive costs. The relative importance between the center's predecessors and successors implies that the optimal sequence for a star network can be summarized by a sufficient statistic, that is, the number of the center's successors.[4] Let m be the number of the center's successors, with $0 \leq m \leq n-1$. Thus, the center has $n-1-m$ predecessors; if any of them shirks, then the center and all his successors shirk accordingly under the optimal reward scheme. Similarly, if the center shirks, then all his successors shirk as well. In contrast, the center's successors cannot trigger anyone to shirk because their actions are unobservable. Define $V^*(m)$ as the total rewards to the agents under the optimal reward scheme when the m most important peripheral agents move after the center. Thus, by Lemma 1, $V^*(m)$ equals

$$\underbrace{\sum_{i=1}^{n-1-m} \frac{1}{p(I) - p(\{j|j < n-m\}\backslash\{i\})}}_{\text{rewards to the predecessors}} + \underbrace{\frac{1}{p(I) - p(\{j|j < n-m\})}}_{\text{reward to the center}} + \underbrace{\sum_{i=n-m}^{n-1} \frac{1}{p(I) - p(I\backslash\{i\})}}_{\text{rewards to the successors}}.$$

To find the optimizer m^*, we compare $V^*(m)$ with $V^*(m+1)$; the difference between the two items is the marginal impact of an additional successor on the total rewards. By direct calculation, for any integer m with $0 \leq m \leq n-2$, $V^*(m+1) - V^*(m)$ equals

$$\begin{aligned} &\sum_{i=1}^{n-2-m} \left[\frac{1}{p(I) - p(\{j|j < n-m-1\}\backslash\{i\})} - \frac{1}{p(I) - p(\{j|j < n-m\}\backslash\{i\})} \right] \\ &+ \frac{1}{p(I) - p(\{j|j < n-m-1\})} - \frac{1}{p(I) - p(\{j|j < n-m\})} \\ &+ \frac{1}{p(I) - p(I\backslash\{n-m-1\})} - \frac{1}{p(I) - p(\{j|j < n-m-1\})}. \end{aligned} \tag{2}$$

The sum of the first two lines in (2) is the change in rewards for the center and his $n-2-m$ predecessors. Since p is increasing, this value is negative. Intuitively, as the center has more successors, his action and his remaining predecessors' actions become more transparent, and thus, these agents' incentive costs are lower. In this regard, we call the negative of the first

[4] Note that the relative order between the center's predecessors or successors does not affect their incentive costs, since each agent's action is equally transparent for the predecessors and successors, respectively.

two lines the marginal benefit (MB) of additional successors. Formally, we define

$$MB(m) := \sum_{i=1}^{n-2-m}\left[\frac{1}{p(I)-p(\{j|j<n-m\}\backslash\{i\})}-\frac{1}{p(I)-p(\{j|j<n-m-1\}\backslash\{i\})}\right] \\ +\frac{1}{p(I)-p(\{j|j<n-m\})}-\frac{1}{p(I)-p(\{j|j<n-m-1\})}.$$

In contrast, the last line in (2) is positive, which is the increase in the new successor's reward, noting that his action becomes less transparent. Thus, we call the sum of the two terms the marginal cost (MC) of additional successors. Formally, we define

$$MC(m) := \frac{1}{p(I)-p(I\backslash\{n-m-1\})}-\frac{1}{p(I)-p(\{j|j<n-m-1\})}.$$

It follows from Corollary 1 that $MB(m)$ is decreasing in m, as the marginal benefit of action transparency is diminishing. On the other hand, note that each new successor of the center is less important than the current ones, meaning that his new reward is higher than that of any other successor. Moreover, the transparency of the new successor's original action (when he was the center's predecessor) is increasing in m; in turn, his original reward is decreasing in m. Together, we have that $MC(m)$ is increasing in m. To summarize,

Lemma 5. *$MB(m)$ is decreasing in m, whereas $MC(m)$ is increasing in m.*

Lemma 5 ensures that there exists a unique optimizer m^* (either an interior solution or a corner solution). Therefore, the optimal sequence is essentially unique and can be succinctly characterized by an integer m^* which is given by

$$m^* := \min\{m|MB(m) \leq MC(m)\}. \tag{3}$$

It follows that one can easily pin down m^* by increasing m one by one from 0 until the first time when $MB(m) \leq MC(m)$. Formally,

Proposition 2. *If g is a star network, then the optimal sequence π^* satisfies that the center has m^* successors, each of them is more important than the center's predecessors, where m^* is given by (3) with $0 \leq m^* \leq n-2$. In particular, if all the agents are equally important, then $1 \leq m^* < n/2$. In contrast, if $[p(I) - p(I\backslash\{n\})] \geq (n-1)[p(I) - p(I\backslash\{n-1\})]$, then $m^* = 0$, where $\{n\}$ is the center and $\{n-1\}$ is the most important peripheral agent.*

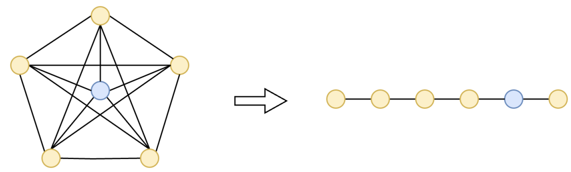
Proposition 2 asserts that the manager (center) never moves the first. Suppose not, then it is profitable to switch the order of the manager and a subordinate. Note that in the new sequence, the subordinate's action is as transparent as the manager's old action, which will be learned by all the other agents, while the manager's action is more transparent than the subordinate's old action which is unobservable. Since any other agent's reward remains the same, the total rewards are strictly lower in the new sequence.

In particular, if the manager is significantly more important than any other agent, then he should move the last, since in this case his predecessors will have relatively low incentive costs due to a significant implicit threat of shirking imposed by him, whereas his successors will be relatively expensive to incentivize as their actions will be unobservable.

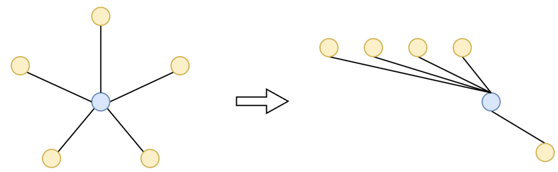
In contrast, if the agents are equally important, then the manager always moves in some interior stage. This is because with identical importance, each agent i's payoff depends only on the cardinality of M_i, $|M_i|$, irrespective of his identity. If the manager moves in an interior stage, his successors can effectively monitor his predecessors through him, as if he served as an *internal information intermediary.* In summary, the manager never moves the first in the optimal sequence; instead, he moves either the last or in some interior stage, depending on his relative importance. This pattern is a direct result of leveraging action transparency.

Furthermore, Proposition 2 indicates that if the agents are equally important, then the number of the manger's successors is (weakly) less than that of his predecessors. This result follows from the diminishing marginal benefit of action transparency. To see the intuition, suppose by contradiction $m^* \geq n/2$, then consider a new sequence π' in which one of his m^* successors, denoted i, moves before the manager. This change has two implications. On the one hand, each action of the manager and his predecessors (in total $n-m^*$ actions) is learned by one fewer agent. On the other hand, agent i's acton is learned by m^* more agents. Note that in π^*, each action of the manager and his predecessors is more transparent than that of agent i. Moreover, in π', the transparency (measured in $|M|$) of each of those $n - m^*$ more transparent actions decreases by 1, whereas that of the less transparent action of i increases by m^*. Since $m^* \geq n - m^*$, the diminishing marginal benefit of action transparency ensures that such an "exchange", or balancing, of transparency is desirable. This effect is analogous to the desire for consumption smoothing with diminishing MRS.

Summary. The analysis in this section reveals that the exogenous network structure plays a significant role in determining the optimal monitoring network. In a clique network, since the agents are fully connected, the optimal monitoring network can simply achieve the most transparent structure characterized by a line. In contrast, a star network has a much sparser structure, having the minimal amount of links needed to connect all the agents. As a result, the optimal monitoring network cannot be determined by directly maximizing transparency;



(a) A clique network (left) and a possible optimal sequence (right). The agents always move sequentially.



(b) A star network (left) and a possible optimal sequence (right). The manager never moves the first.

Figure 1: The optimal sequences for a clique and a star network. The examples in panels (a) and (b) share the same number of agents, n, and the same technology, p. However, in (a), g represents a clique network, while in (b), it represents a star network. In this figure and any subsequent figures, a blue node represents a manager, and a yellow one represents a subordinate. In addition, when illustrating the optimal sequence, we assume that the agents move in a left-to-right order.

instead, it balances two countervailing motives: To enlarge the set of the center's predecessors to increase the number of monitored and less-rewarded agents, and to enlarge the set of the center's successors who monitor the center and his predecessors but are themselves relatively expensive to incentivize. Under the diminishing marginal benefit of action transparency, the optimal monitoring network can be determined by a unique number of the center's successors through a simple algorithm. In particular, when the agents are equally important, the center always has (weakly) fewer successors than predecessors, to balance internal transparency.

Furthermore, the analysis also reveals how an agent's position in the exogenous network affects his role in the optimal monitoring network. Note that in a clique network the agents' positions are symmetric, while in a star network the manager is centrally located and all the subordinates are in peripheral positions. As a consequence, the optimal sequence of a clique network depends only on the importance ranking of the agents, while in a star network the manager must move after some subordinate to facilitate peer monitoring. Moreover, in both networks, more important agents will move in later stages to enhance the implicit threat of shirking. These findings are illustrated in Figure 1.

5 Multiple teams

We now turn to the case in which there are multiple teams, all of them are characterized by either clique networks or star networks. Our primary goal is to further investigate how the connectivity structure of these teams affects the optimal monitoring network. This analysis can yield valuable insights into understanding how the internal information within large-scale organizations depends on their organizational structures. We consider two typical structures: a *flat organization* in which the managers are interconnected, and a *hierarchical organization* in which the managers are connected through a top manager. Within the first class, we also distinguish between a *non-divisional network* and a *divisional network*, which only differ in whether a manager is also linked to the subordinates of other teams.

For tractability, we hereafter assume that each agent is equally important to the project. Thus, the success probability is given by an increasing and convex function $p(\cdot)$, where the input is the number of agents who work. Moreover, we can now measure the transparency of each agent i's action by $|M_i|$. For ease of exposition, we proceed to study the aforementioned network structures in approximately descending order of network density.

5.1 Non-divisional network: Dense core-periphery

Assume that each team is characterized by a clique network and each manager is connected to all the other agents. We call such a network a *dense core-periphery* network, denoted g_{dcp}. Figure 2 depicts an example of g_{dcp}. Note that dense core-periphery networks exhibit a nice property for solving the optimal sequence. Specifically, g_{dcp} has an *open Hamilton walk*, i.e., a walk that involves every node in the network exactly once. For example, as illustrated in Figure 2, such a walk can begin with any manager and visit every subordinate in the same team exactly once, then turn to a different manager, repeating the process of visiting all the remaining managers and their subordinates until reaching the final unvisited agent. Such a sequence is clearly optimal as it generates the most transparent internal information; indeed, it aligns with an optimal sequence when the agents are fully connected. Formally,

Proposition 3. *If g is a dense core-periphery network, then the optimal sequence $\pi^*(g_{dcp})$ is an open Hamilton walk in g, in which there can be at most two adjacent managers.*

Proposition 3 implies that the optimal monitoring network is essentially characterized by an alternating pattern between the managers and their subordinates. Such a pattern allows the principal to leverage the high density of g_{dcp}, especially the high degrees of the managers, to establish a line-structured monitoring network, thereby maximizing internal information. Without loss of generality, we focus on the open Hamilton walk that starts with the manager from the largest team and sequentially visits every subordinate in the same team, then visits the remaining teams in descending order of team size. Therefore, the number of subordinates across managers is decreasing along the sequence.

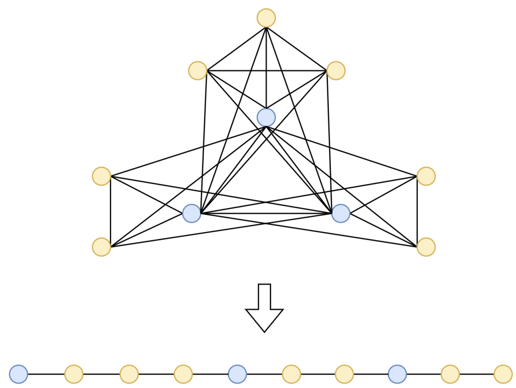


Figure 2: Example of a dense core-periphery network (top) and a resulting optimal sequence (bottom).

5.2 Non-divisional network: Core-periphery

Assume that each team is characterized by a star network and each manager is connected to all the other agents. Such a network is therefore a *core-periphery* network, denoted g_{cp}. Figure 3 depicts an example of g_{cp}. In contrast to a dense core-periphery network, a core-periphery network has much sparser within-team connections, as each pair of subordinates is not connected. It can be shown that there does not exist an open Hamilton walk in such a network, thus characterizing the optimal sequence is much more challenging than in dense core-periphery networks. However, we can derive some important properties of the optimal sequence, which are presented in the proposition below.

Proposition 4. *If g is a core-periphery network, then the optimal sequence $\pi^*(g_{cp})$ can be characterized by a series of positive integers $\{s_q\}_{q=1}^{|T|+1}$, such that s_q subordinates move before the q-th manager for $q \leq |T|$, and $s_{|T|+1}$ subordinates move after the last manager, satisfying $\sum_{q=1}^{|T|+1} s_q = \sum_{t\in T} f_t$. Moreover, $s_q \geq 1$ and is nonincreasing in q for all $1 \leq q \leq |T|+1$.*

Proposition 4 indicates that the optimal monitoring network of a core-periphery network also exhibits a similar alternating pattern between managers and subordinates. Specifically, any two managers cannot be adjacent, and any manager cannot move the first or the last in the entire sequence. This pattern also follows from leveraging action transparency. Suppose by contradiction there are two adjacent managers in the optimal monitoring network, then by the pigeonhole principle and the structure of g_{cp}, there are multiple subordinates moving between two other managers and parallel to each other. Now, move one of the subordinates to between the two adjacent managers. Doing so essentially transforms simultaneous moves into sequential moves, thereby enhancing peer monitoring. Thus, we obtain a contradiction. Similarly, a manager cannot be either the first mover or the last mover in the sequence.

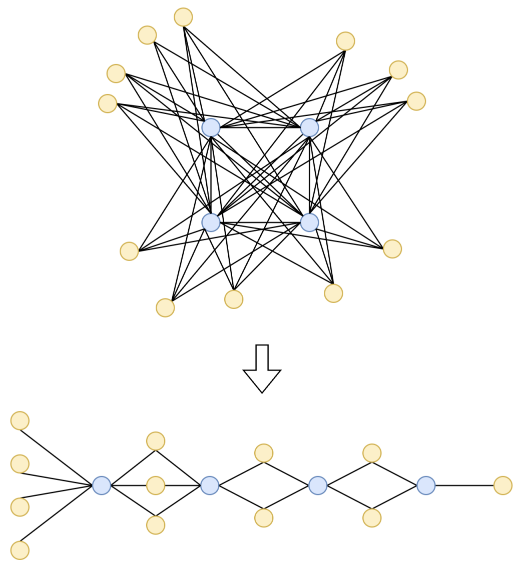


Figure 3: Example of a core-periphery network (top) and a possible optimal sequence (bottom).

Therefore, in the optimal monitoring network, every manager is positioned between two groups of subordinates. Within each group, agents are unable to see each other, while agents in the later group can learn each agent's action in the earlier group through the manager. It follows that such a subgraph has a star network structure. Since managers and subordinates alternate in the optimal sequence, by an analogous argument as in Section 4.2, the number of subordinates across managers is nonincreasing along the optimal sequence, as characterized by Proposition 4. This pattern is illustrated by Figure 3.

Summary. Propositions 3 and 4 together reveal that the optimal monitoring network of a non-divisional network, including dense core-periphery and core-periphery networks, exhibits an alternating pattern between managers and subordinates, with the number of subordinates across managers nonincreasing along the optimal sequence. The alternating pattern follows directly from leveraging the non-divisional structure to facilitate peer monitoring, while the monotonicity of the number of subordinates between managers is driven by the diminishing marginal benefit of action transparency.

5.3 Divisional network: Connected-cliques

Assume that each team is characterized by a clique network and all the managers are interconnected. We call such a network a *connected-cliques* network, denoted g_{cc}. Figure 4 depicts an example of g_{cc}. In contrast to the above non-divisional networks, since any two teams are connected only through their managers, Lemma 2 implies that the managers form a line in

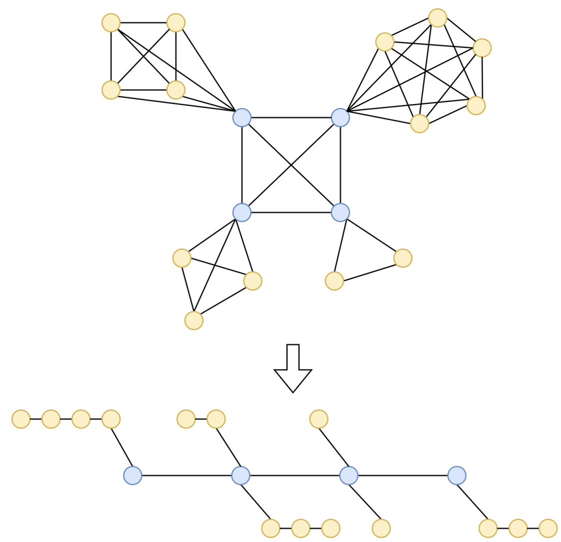


Figure 4: Example of a core-periphery network (top) and a possible optimal sequence (bottom).

the optimal monitoring network, thereby serving as the internal information intermediaries of the organization. Moreover, since every team is a clique, in the optimal sequence, within each team the agents also form a line. For ease of exposition, provided there is no confusion, we hereafter call a subordinate who moves before his manager a *predecessor*, and who moves after his manager a *successor*. In addition, we say that a team is a *type-I* (*type-III*) team if all the subordinates are predecessors (successors), and a *type-II* team if it is neither type-I nor type-III. We also say that team t moves before team t' if t's manager moves before t''s. The lemma below characterizes the optimal relative order among teams of the same type.

Lemma 6. *If g is a connected-cliques network, then along the optimal sequence $\pi^*(g_{cc})$: (i) the size of type-I team, if it exists, is nonincreasing; (ii) both the numbers of predecessors and successors of type-II team, if it exists, are decreasing; (iii) the size of type-III team, if it exists, is nondecreasing; (iv) all type-I teams move before all type-III teams, if they exist.*

Lemma 6 results from leveraging action transparency. Note that if team t is type-I, then its subordinates' actions can be learned by its manger l_t and those who can learn l_t's action. Conversely, if team t is type-III, then its subordinates can learn l_t's action and those can be learned by l_t. Therefore, assigning a larger type-I team to an earlier stage can improve the transparency of more actions, while assigning a larger type-III team to a later stage enables a larger number of agents to monitor more agents. Moreover, suppose a type-I team moves after a type-III team, then it is clearly desirable to switch the order of the two teams, since it reduces at least the rewards to the subordinates of that type-I team.

Regarding type-II teams, consider two such teams t and t', such that t moves before t' in

$\pi^*(g_{cc})$. Suppose there are (weakly) more successors in t' than in t, then reduce a successor in t and increase a successor in t', such that agents within each team also form a line; denote the new sequence π'. Let i be the immediate successor of l_t within t in $\pi^*(g_{cc})$, and j be the immediate successor of $l_{t'}$ within t' in π'. Note that the net change in action transparency between $\pi^*(g_{cc})$ and π' in t is as if i's transparency increased from $|M_i^*|$ to $|M'_{l_t}|$, and that in t' is as if $l_{t'}$'s transparency decreased from $|M^*_{l_{t'}}|$ to $|M'_j|$. In addition, the actions of all the managers and predecessors between t and t' are more transparent in π' since there are more successors in t', and the action of any other agent is as transparent as before. Since there are more successors in t' than in t in $\pi^*(g_{cc})$, and the total number of successors in t' and t are unchanged, we have $|M_i^*| < |M'_j|$ and $|M'_{l_t}| > |M^*_{l_{t'}}|$. Thus, the increase in transparency in t exceeds the decrease in transparency in t', a contradiction. This also implies that a type-III team must move after any type-II team with more successors than it.

Next, suppose there are (weakly) more predecessors in t' than in t. From the above, t has more successors than t'. Switch the order of t and t'. As such, the actions of the predecessors in t' become more transparent, while those of the predecessors in t are now less transparent. Moreover, the actions of all the managers and predecessors between t and t' become more transparent given that t has more successors than t'. Since there are more predecessors in t' than in t, such an change increases the transparency of a larger number of actions in t' more than it decreases the transparency of actions in t, a contradiction. This also implies that a type-III team must move after any type-II team with fewer successors than it.

Then, combining the above paragraphs, we have that any type-III team must move after all type-I and type-II teams, if they exist. Moreover, to enhance transparency, in the optimal sequence, clearly, the first team must be a type-I team, and the last team must be a type-III team. To summarize, we have the following proposition.

Proposition 5. *If g is a connected-cliques network, then along the optimal sequence $\pi^*(g_{cc})$: (i) the first team is a type-I team and the last team is a type III-team; (ii) all type III-teams move after all type-I and type-II teams; (iii) the relative order among teams of the same type satisfies Lemma 6. Furthermore, if the size of any team is at least half the size of any other team, then all type-II teams, if they exist, move between type-I teams and type-II teams. In particular, if each team has the same size, then there exists at most one type-II team.*

Proposition 5 indicates further that if the heterogeneity of team sizes is relatively small, then all type-II teams, if they exist, should move between type-I teams and type-III teams, such that in the optimal sequence, it is successively type-I, type-II, and type-III teams. This is because if a type-I team moves after a type-II team, the relatively small difference in team sizes allows for enough flexibility to improve the transparency by switching either the order or the type of these two teams, so that some agents' actions are strictly more transparent.

Furthermore, if the team sizes are identical, then one can fully characterize the optimal sequence by a simple algorithm. To illustrate, let m_t be the number of successors in team t, and $\sum m_t$ be the total number of successors. Define $MB(\sum m_t)$ as the marginal benefit of increasing $\sum m_t$, which equals the decrease in rewards for the managers and predecessors in teams before the new successor's team. Define $MC(\sum m_t)$ as the marginal cost of increasing $\sum m_t$, which equals the increase in rewards for the new successor's team, as if the manager moved from his initial position to the current position of his immediate successor within his team. In the appendix, we show that $MB(\sum m_t)$ is always decreasing, whereas $MC(\sum m_t)$ is decreasing within any team but increasing across teams. Thus,

Corollary 3. *Suppose each team has the same size, then $\pi^*(g_{cc})$ can be fully characterized through the following algorithm:*

1. *Set each team t a type-I team, i.e., $m_t = 0$, $t \in T$.*
2. *From the last team to the first team, increase m_t one by one until the first team such that $MB(\sum m_t) < MC(\sum m_t)$, and denote this team t^*.*
3. *Within team t^*, search the optimal $m^*_{t^*}$ that minimizes the total rewards.*

5.4 Divisional network: Connected-stars

Assume that each team is characterized by a star network and all the managers are interconnected. We call such a network a *connected-stars* network, denoted g_{cs}. Figure 5 depicts an example of g_{cs}. Similar to connected-cliques networks, in the optimal monitoring network of a connected-stars network, the managers also form a line. We also define analogously the concepts of predecessor and successor, as well as type-I, type-II and type-III teams. Using a similar argument as in Section 5.3, we have the following proposition.

Proposition 6. *If g is a connected-stars network, then in the optimal sequence $\pi^*(g_{cs})$, the foremost segment consists of a possibly empty set of type-I teams, organized in descending order of size. Following this, there may be at most one type-II team, succeeded by another possibly empty set of type-III teams, organized in ascending order of size. Moreover, the sets of predecessors and successors are both nonempty.*

Proposition 6 indicates that in the optimal monitoring network, all the subordinates can be divided into two groups: the earlier group consists of only predecessors, whereas the later

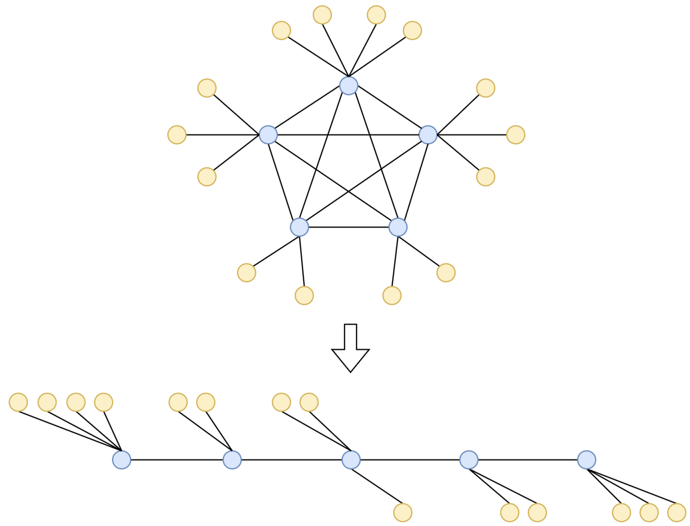


Figure 5: Example of a connected-stars network (top) and a possible optimal sequence (bottom).

group consists of only successors. To see the intuition, consider changing a predecessor into a successor. Note that the marginal benefit of an additional successor is higher within a later team, since a successor in a later team can learn more preceding actions. Note too that the marginal cost of an additional successor equals the increase in reward for the new successor. Since a successor's action is unobservable, and a predecessor's action is more transparent if he is in an earlier team, the marginal cost is lower within a later team. This implies that all predecessors move before all successors. It follows that the optimal monitoring network has at most one type-II team, which is between type-I and type-II teams, if they exist.

Furthermore, when both type-I and type-II teams exist, the optimal monitoring network exhibits a "V-shape". As illustrated in Figure 5, T can be basically divided into two groups: in the earlier group, each team is a type-I team, organized in descending order of size, and in the later group, each team is a type-III team, organized in ascending order of size. Note that each subordinate in type-III teams effectively monitors those in type-I teams through the managers. By allocating larger teams towards either end of the sequence, one can enhance the monitoring of a greater number of agents through an increased number of monitors.

While Proposition 6 provides only a partial characterization of the optimal sequence, it rules out most suboptimal sequences. Specifically, given the number of teams $|T|$, there are in total $|T|!$ possible permutations of teams. However, the number of the permutations that satisfy Proposition 6 is around $\sum_{k=0}^{|T|} C(|T|, k) = 2^{|T|}$,[5] which is of lower order of $|T|!$ for a

[5]Partition the stars into two groups, in the first each subordinate is a predecessor, whereas in the second each subordinate is a successor. Let k be the number of stars in the first group. Then, the relative order of teams is given by Proposition 6. The number of possible permutations is thus $2^{|T|}$.

large $|T|$. Moreover, if the relative order among teams in $\pi^*(g_{cs})$ is known, then $\pi^*(g_{cs})$ can be fully characterized by a simple algorithm as in Corollary 3. Define analogously $MB(\sum m_t)$ and $MC(\sum m_t)$. As discussed previously, $MB(\sum m_t)$ again stems from the improvement of transparency of all the actions that can be learned by the new successor, while $MC(\sum m_t)$ is instead given by the increase in reward for the new successor. Similar to a single star, we have that $MB(\sum m_t)$ is decreasing whereas $MC(\sum m_t)$ is increasing in $\sum m_t$. Thus,

Corollary 4. *Suppose the managers move according to the optimal sequence $\pi^*(g_{cs})$, then $\pi^*(g_{cs})$ can be fully characterized through the following algorithm:*

1. *Set each team t a type-I team, i.e., $m_t = 0$, $t \in T$.*
2. *From the last team to the first team, increase m_t one by one until the first time when $MB(\sum m_t) \leq MC(\sum m_t)$. The resulting sequence is exactly $\pi^*(g_{cs})$.*

When the number of teams $|T|$ is relatively small, it is convenient to characterize $\pi^*(g_{cs})$ by applying the algorithm in Corollary 4 to the set of undominated sequences characterized by Proposition 6. In contrast, when both n and $|T|$ are large, there are still numerous feasible ways to group the teams, making such a method impractical. Nevertheless, we can show that under some mild conditions—the following Assumption 1 for example—the total number of successors $\sum m_t^*$ in $\pi^*(g_{cs})$ is of lower order of n. That is, the set of successors amounts for only a small fraction of the organization. This enables us to significantly reduce the number of possible combinations for type-III teams, thereby eliminating numerous suboptima.

Assumption 1. *For any positive integers $m \leq n$, we have*

$$\frac{1}{m}[p(n) - p(n-m)] \geq K[p(n) - p(n-1)]$$

for some constant $K > 0$.

Assumption 1 requires that the marginal productivity of the last unit of effort cannot be infinitely higher than the average marginal productivity of any number of efforts; otherwise, $MC(\sum m_t)$ might be relatively low for a broad range of $\sum m_t$, such that a large fraction of subordinates are successors. Moreover, in a connected-cliques network, if each team's size is small relative to the population n, then Assumption 1 also ensures that the total number of successors is of lower order of n. Formally,

Proposition 7. *Given Assumption 1, in the optimal sequence of a connected-stars network, the number of successors is bounded above by some number of order $\sqrt{n}$, as $n \to \infty$. Also, in* *a connected-cliques network, as* $n \to \infty$*, if the size of each team is bounded by some constant, then the number of successors is also bounded above by some number of order* $\sqrt{n}$*.*

We sketch the proof of Proposition 7 here by focusing on a connected-stars network g_{cs}. The argument for a connected-cliques network with bounded team sizes is analogous. First, we shall show that given any number of successors $\sum m_t$, the marginal benefit $MB(\sum m_t)$ in g_{cs} is lower than the marginal benefit $MB(m)$ in a single star with a same n, if in g_{cs}, the number of the agents who move before the manager of the first non-type-I team is equal to the number of the center's predecessors in the star. Intuitively, since g_{cs} is denser than star, the agents will have more transparent actions in g_{cs} than in the star. Under the diminishing marginal benefit of action transparency, $MB(\sum m_t)$ is less than the corresponding $MB(m)$, and thus, there will be fewer successors in g_{cs} than in the star. Next, we shall show that the number of successors in the star is of order $\sqrt{n}$ given Assumption 1. This is because on the one hand, the marginal benefit of action transparency is diminishing. On the other hand, a successor is relatively expensive to incentivize due to his unobservable action. As a result, $MB(m)$ will soon be exceeded by $MC(m)$, leading to a small m^* and thus a small $\sum m_t^*$.

Proposition 7 offers an important insight in understanding the role of peer monitoring. That is, whereas peer monitoring is beneficial in cutting the agents' incentive costs, within a divisional network, as well as a star network, the monitors themselves are relatively costly to incentivize, as there are few or no individuals who will monitor the monitors. Consequently, the number of monitors accounts for only a small fraction of the entire population.

Summary. The analysis in Sections 5.3 and 5.4 reveals that the optimal monitoring network of a divisional network is prominently different from that of a non-divisional network. Note that a non-divisional network will yield a monotone structure of monitoring network, in the sense that managers and subordinates alternate in the optimal sequence and the number of subordinates across managers is nonincreasing. In contrast, a divisional network will yield a non-monotone structure of monitoring network, in the sense that larger teams are assigned to either end of the optimal sequence. Specifically, subordinates in earlier stages are effectively monitored by subordinates in later stages, through the managers who serve as the internal information intermediaries. Moreover, in a divisional network, monitors are relatively costly to incentivize, since their actions are unobservable; thus, there will be a small set of monitors clustering in the end of the optimal sequence.

5.5 Hierarchical networks

We now turn to a hierarchical network, in which there is a special team in T, which consists of a single agent $h \in I$, referred to as the *top manager*, such that the manager of any other team is directly linked to h, and there are no direct links between any two agents $i, j \neq h$, who are from different teams. Similarly, we study networks in which all teams, except for the special team, are characterized by either cliques or stars, referred to as a *hierarchical-cliques* network and a *hierarchical-stars* network, respectively.

We first consider a hierarchical-cliques network, denoted g_{hc}. Figure 6 depicts an example of g_{hc}. We define type-I, type-II and type-III teams in the same way as for connected-cliques networks. Given the structure of g_{hc}, by Lemma 2, every manager moves either before h or after h. The next proposition characterizes important properties for the optimal monitoring network of a hierarchical-cliques network.

Proposition 8. *If g is a hierarchical-cliques network, then in the optimal sequence $\pi^*(g_{hc})$, every team before h is a type-I team, and every team after h is a type-III team; both the sets of type-I and type-III teams are nonempty. Moreover, let x and y be the numbers of agents before and after h, respectively, and let t be the smallest team after h. Then, $x > y - 2f_t$.*

Proposition 8 indicates that the optimal monitoring network of g_{hc} has a similar structure as a single star: the top manager is located in the central position, moving after a nonempty set of type-I teams and before a nonempty set of type-III teams; thus, the agents in type-III teams can effectively monitor those in type-I teams through the top manager, who serves as the internal information intermediary.

Furthermore, Proposition 8 asserts that there are relatively few agents moving after the top manager: the number of these agents is less than around half of the population. Similar to a single star, if there are many agents after the top manager, then there will be few agents before him, whose actions are relatively transparent. With the diminishing marginal benefit of action transparency, it is profitable to move the smallest team after the top manager to before him and make it a type-I team, because the benefit of increasing the transparency of those less transparent actions outweighs the cost of reducing the transparency of those more transparent ones. That is, the distribution of agents across the top manager is driven by the desire for balancing internal transparency.

Now, we turn to a hierarchical-stars network, denoted g_{hs}. Figure 7 depicts an example of g_{hs}. The next proposition shows that the optimal monitoring network of a hierarchical-stars network has a similar star structure, with relatively few agents after the top manager.

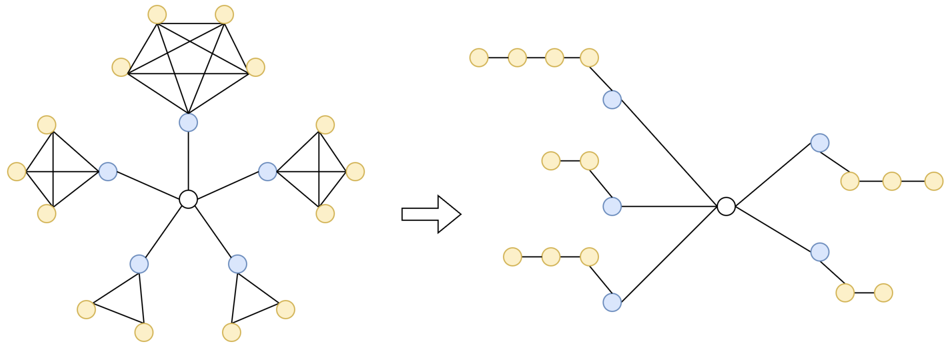


Figure 6: Example of a hierarchical-cliques network (left) and a possible optimal sequence (right).

Proposition 9. *If g is a hierarchical-stars network, then in the optimal sequence $\pi^*(g_{hs})$, either (i) every team before h is a type-I team, and every team after h is a type-II or type-III team, or (ii) every team before h is a type-I or type-II team, and every team after h is a type-III team; both the sets of teams before and after h are nonempty. Let m_t^* be the number of successors in each team t. For any two type-II teams t' and t'' with $f_{t'} < f_{t''}$, $m_{t'}^* \leq m_{t''}^*$. Moreover, let x be the number of managers and predecessors before h, and y be the number of managers and successors after h, and let t be the smallest team after h. Then, $x > y - 2f_t$.*

Unlike in a hierarchical-cliques network, in a hierarchical-stars network, there may exist some type-II teams in the optimal monitoring network, in which case, all type-II teams must be either before or after the top manager. This is due to the similar reason that a successor should not move before a predecessor, as this could hinder peer monitoring. Moreover, the number of successors in a type-II team is nondecreasing in its size. This again follows from the desire for balancing internal transparency. Specifically, if a larger type-II team has fewer successors than a smaller type-II team, then there are at least two more predecessors in the larger team, and each predecessor in the larger team has a more transparent action than his counterpart in the smaller team. This implies that it is profitable to increase a successor in the larger team, while simultaneously decrease a successor in the smaller team.

Given the star structure of the optimal monitoring network in a hierarchical network, it is reasonable to expect that the set of successors (monitors) still constitutes a small fraction of the entire organization as it expands. This intuition holds true if the size of each team is bounded by some constant. Formally,

Proposition 10. *Given Assumption 1, suppose as $n \to \infty$, the size of each team is bounded by some constant in both a hierarchical-cliques and a hierarchical-stars network. Then, in the optimal sequence of each network, the number of managers and successors after h is bounded above by some number of order $\sqrt{n}$, as $n \to \infty$.*

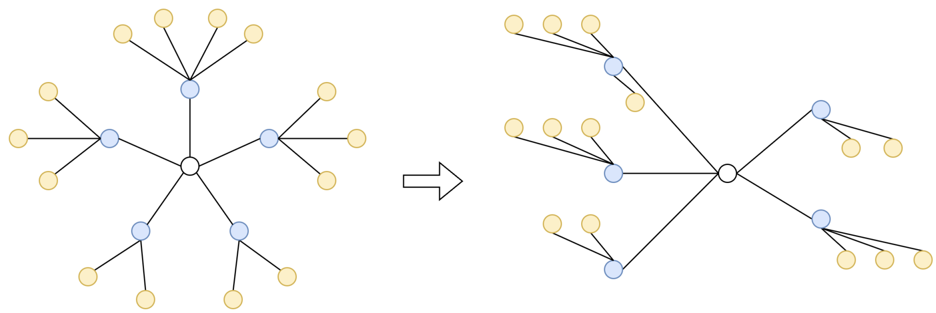


Figure 7: Example of a hierarchical-stars network (left) and a possible optimal sequence (right).

The intuition is similar to that of Proposition 7. In a hierarchical network, all successors after the top manager effectively monitor all predecessors before the top manager, reducing the incentive costs of those monitored. However, those monitors are significantly expensive to incentivize, as their actions are much less transparent. Thus, there is only a small fraction of agents who serve as monitors, when each team is relatively small; otherwise, it is desirable to balance internal transparency by moving a team after the top manager to before the latter, and making it a type-I team, so that the subordinates in this team become monitored.

Summary. The analysis in this section reveals that the optimal monitoring network within a hierarchical network resembles a star, with the top manager positioned centrally. A group of agents following the top manager effectively monitor those preceding him through the top manager and team managers. With the desire for balancing internal transparency, there will be relatively few agents after the top manager, especially when n is large.

6 Comparison of networks

The previous analysis demonstrated how different network structures may result in different optimal monitoring networks and thus different internal information. It is natural to further investigate which network structure the principal would prefer. In this section, we compare the networks studied in Section 5 based on two criteria: total cost and stability, which will be formally defined later. The purpose of this analysis is to provide insights into identifying effective and efficient organization structures for leveraging peer monitoring.

To facilitate comparison, in this section, we assume homogeneous team sizes: $f_t \equiv \hat{f}$ for some integer $\hat{f} \geq 2$, except for the special team with top manager in a hierarchical network.[6] We also impose additional structure on the production function p, such that for arbitrarily

[6] In this section, the top manager is not considered as one of the agents; thus, the principal does not need to provide incentives to him. The top manager will work if and only if he does not see anyone shirking.

large n, $p(n) \equiv p_0$, $p(n-1) \equiv p_1$, $p(n-2) \equiv p_2$, $\dots$, where $1 \geq p_0 > p_1 > p_2 > \cdots \geq 0$ are a sequence of constants; that is, for any fixed n, the i-th highest output level, $p(n+1-i)$, is given by the i-th number in the sequence $\{p_0, p_1, p_2, \dots\}$. To ensure that complementarity still holds, we assume that for $n' = 0, 1, \dots$, $p_{n'} - p_{n'+1}$ strictly decreases in n'.

6.1 Ranking networks by total cost

Given the set of agents I, if adding links to g were costless, then by Corollary 1, the principal would always prefer a supergraph of g to g. In reality, however, adding and maintaining links between agents are often expensive or even infeasible, especially for those indispensable for a desired monitoring network. Thus, to offer an interesting and practical comparison among different networks, we shall measure the cost of implementing a monitoring network, which is jointly determined by g and π. To implement a particular monitoring network of g, it is possible that not all links in g are necessary; that is, some links are essential while the rest are not. In this regard, we modify a terminology in graph theory, the *transitive reduction*, to characterize the "minimal" network needed to implement a given monitoring network:

Definition 1. *Given a sequence π, the transitive reduction of g is a network $\tilde{g}(\pi; g) \subseteq g$, $\tilde{g}$ for short, such that (i) $v^*(\pi)$ is identical under g and $\tilde{g}$, and (ii) there does not exist another network $g' \neq \tilde{g}$, which satisfies (i) but with fewer links.*

For example, as illustrated in Figure 1, to implement the optimal monitoring network of a clique network, the transitive reduction $\tilde{g}$ is a line. We assume that each link in $\tilde{g}$ incurs a maintenance cost $c > 0$. Then, we define the *total cost* of network g as the minimized sum of the costs of maintaining links in $\tilde{g}$ and the total rewards under the resulting optimal reward scheme. Formally, the total cost is given by

$$C(g) \coloneqq \min_{\pi} \left[V^*(\pi; g) + \frac{c|\tilde{g}(\pi; g)|}{2} \right],$$

where $V^*(\pi; g)$ is the minimized total rewards given π and g. Note that the cardinality of an undirected network, $|\tilde{g}|$, equals twice the number of links in $\tilde{g}$, as $ij \in \tilde{g} \Leftrightarrow ji \in \tilde{g}$. Note too that the cost-minimizing sequence is not necessarily the optimal sequence as defined before, since the principal now faces a trade-off between link maintenance costs and total rewards. To distinguish between the two sequences, we call the former an *efficient sequence*.

We also assume that c is sufficiently small, such that an efficient sequence always results in a *connected* transitive reduction, i.e., every two agents are connected by some path in $\tilde{g}$. The next proposition ranks the networks studied in Section 5 in terms of total cost.

Proposition 11. *Let* $G := \{g_{dcp}, g_{cp}, g_{cc}, g_{cs}, g_{hc}, g_{hs}\}$*. For sufficiently small* $c > 0$*:*

(i) $C(g_{dcp}) < C(g)$ *for all* $g \in G, g \neq g_{dcp}$*, while* $C(g_{hs}) > C(g)$ *for all* $g \in G, g \neq g_{hs}$*;*

(ii) $C(g_{cc}) < \min\{C(g_{cs}), C(g_{hc})\}$*, and* $C(g_{cp}) < C(g_{cs})$*;*

(iii) $C(g_{cp}) < C(g_{cc}) < C(g_{hc})$ *when* $\hat{f}$ *is fixed and* $|T|$ *is sufficiently large, while* $C(g_{cc}) < C(g_{hc}) < C(g_{cp})$ *when* $|T|$ *is fixed and* $\hat{f}$ *is sufficiently large.*

Statements (i) and (ii) summarize a number of intuitive general properties of total cost. If g is a supergraph of g', so that some agents are strictly better connected in g than in g', then $C(g) < C(g')$. This is reflected by that in G, g_{dcp} is the least costly network and g_{hs} is the most costly one, as well as that $C(g_{cc}) < C(g_{cs}) < C(g_{hs})$, $C(g_{cc}) < C(g_{hc}) < C(g_{hs})$, and $C(g_{cp}) < C(g_{cs}) < C(g_{hs})$, as indicated by (i) and (ii). The reason why g must bear a lower total cost, even though links in its transitive reduction may be more costly, is twofold. On the one hand, g can yield more internal information and thus requires lower total rewards for full effort. On the other hand, the larger number of links in g typically does not add to link maintenance costs in its transitive reduction; for example, the transitive reduction of g_{cc} and that of g_{cs} share the same number of links. Even if it does, the definition of total cost implies that it can only be in the principal's interest to implement a transitive reduction in g with more links. For example, between g_{cp} and g_{cs}, the principal could always choose the same sequence and pay lower total rewards in g_{cp}. Thus, if an efficient sequence in g_{cp} leads to a transitive reduction that is more than minimally connected, it must be that having more links reduces total rewards more than their maintenance costs.

More interestingly, statement (iii) highlights a novel trade-off between global and local connections. To illustrate, we compare g_{cp} with g_{cc} and g_{hc}, noting that g_{cp} has the maximal inter-team links among the networks in G, while g_{cc} and g_{hc} have every possible intra-team link. While $C(g_{cc}) < C(g_{hc})$ for all n, how they compare asymptotically to $C(g_{cp})$ depends on the dimension along which the organization expands: The principal will ultimately prefer the global connection provided by g_{cp} as the number of teams increases ($|T| \to \infty$), but prefer the local connection provided by g_{cc} or g_{hc} as the size of each team increases ($\hat{f} \to \infty$).

The rationale behind this insight lies in understanding how the principal may create the maximal internal information with the minimal link maintenance costs, as $|T| \to \infty$ and as $\hat{f} \to \infty$, respectively. First, when $\hat{f}$ is fixed and $|T| \to \infty$, we compare g_{cc} and g_{cp}. Since an efficient sequence of each $g \in G$ results in a connected transitive reduction, $\pi^*(g_{cc})$ is indeed an efficient sequence of g_{cc}, as it yields a minimally connected monitoring network with $n-1$ links. Since team size is identical, Proposition 5 implies that in $\pi^*(g_{cc})$, all subordinates in earlier stages are predecessors and form a line in own team, and the remaining subordinates are successors and form a line in own team. Given the structure of p, successors will require remarkably higher rewards than others. It follows that in $\pi^*(g_{cc})$, the number of teams with successors is of lower order of $|T|$, as $|T| \to \infty$. Let T' and I' denote the set of these teams, and the set of managers and successors within these teams, respectively.

Now, construct a sequence π' for g_{cp} by alternating managers and subordinates one by one in all teams to form a line, and assigning each remaining subordinate to before the first manager; the length of this line is $2|T|$. We compare the total rewards between g_{cc} and g_{cp} in three steps. First, the last $|I'|$ agents in the line of g_{cp} require lower rewards than those in I' of g_{cc}, since the former agents' actions are more transparent. Second, note that in $\pi^*(g_{cc})$, the first team and all managers not in I' together form a line with a length $\hat{f} + |T| - |T'|$. Meanwhile, in π', the next-to-last $\hat{f} + |T| - |T'|$ agents remain in the line, as both $|T'|$ and $|I'|$ are small relative to $|T|$. Thus, the $\hat{f} + |T| - |T'|$ agents in π' also require lower rewards than their counterparts in $\pi^*(g_{cc})$. Finally, among the remaining agents in $\pi^*(g_{cc})$, the first agent in the second team has the most transparent action, which can be learned by less than $|T| + |I'|$ agents, whereas in π', each remaining agent's action can be learned by $2|T|$ agents; thus, the latter agents require lower rewards. Together, we have that the total rewards are lower in π' than in $\pi^*(g_{cc})$. Since π' also yields a monitoring network with $n-1$ links, and is not necessarily an efficient sequence of g_{cp}, g_{cp} must incur a lower total cost than g_{cc}. In short, when $|T|$ is large, g_{cp} allows the principal to align agents in a significantly longer line than in g_{cc} or g_{hc}, thereby enhancing peer monitoring with minimal links.

In contrast, when $|T|$ is fixed and $\hat{f} \to \infty$, the principal can create a significantly longer line in g_{cc} or g_{hc} than in g_{cp}. To see this, compare again g_{cc} and g_{cp}. Proposition 5 indicates that the last team in $\pi^*(g_{cc})$ is a type-III team, so any agent outside this team is monitored by at least $1 + \hat{f}$ agents, or equivalently $1/|T|$ fraction of all the agents. On the other hand, it can be shown that as $\hat{f} \to \infty$, in any efficient sequence of g_{cp}, either a substantial fraction of agents move between the first and the last managers, with each agent incurring two links, or an infinitesimal fraction of agents move after the first manager. In the first case, g_{cp} incurs a higher total cost than g_{cc} due to higher link costs. In the second case, g_{cp} again incurs a higher total cost due to higher incentive costs, because in the efficient sequence of g_{cp}, each agent before the first manager is monitored by only an infinitesimal fraction of agents, while his counterpart in $\pi^*(g_{cc})$ is monitored by a $1/|T|$ fraction of agents. In summary, g_{cc} incurs a lower total cost than g_{cp} when $|T|$ is fixed and $\hat{f}$ is sufficiently large.

6.2 Ranking networks by stability

We now examine a second measure for ranking networks, which identifies the largest possible fraction of links that can be severed from network g, while keeping the transitive reduction of g intact. For any connected network g, the *stability* measure $S(g)$ is given by

$$S(g) \coloneqq \frac{|g| - |\tilde{g}(\pi^*; g)|}{|g|},$$

where $\pi^*(g)$ is an optimal sequence for g, and $\tilde{g}(\pi^*; g)$ is the transitive reduction under π^* and g. Note that $S(g) \in [0, 1)$ for any g, and that the higher $S(g)$ is, the more stable g is, as it accommodates a larger fraction of broken links. Our last proposition ranks the networks studied in Section 5 in terms of stability.

Proposition 12. *Let* $G = \{g_{dcp}, g_{cp}, g_{cc}, g_{cs}, g_{hc}, g_{hs}\}$*. Then, we have*

(i) $S(g_{dcp}) > S(g)$ *for all* $g \in G, g \neq g_{dcp}$*, while* $S(g_{hs}) = 0 < S(g)$ *for all* $g \in G, g \neq g_{hs}$*;*

(ii) $S(g_{cc}) > \max\{S(g_{cs}), S(g_{hc})\}$*, and* $S(g_{cp}) > S(g_{cs})$*;*

(iii) $S(g_{cp}) > S(g_{cc}) > S(g_{cs}) > S(g_{hc})$ *when* $\hat{f}$ *is fixed and* $|T|$ *is sufficiently large, while* $S(g_{cs}) < S(g_{cp}) < S(g_{hc}) < S(g_{cc})$ *when* $|T|$ *is fixed and* $\hat{f}$ *is sufficiently large.*

Proposition 12 indicates that the stability ranking among these networks aligns closely with their cost ranking. Statements (*i*) and (*ii*) mean that a denser network g enhances both cost efficiency and stability compared to a sparser network g'. That is, g not only allows the principal to implement a sequence with a lower total cost, but also contains more idle links outside the corresponding transitive reduction for higher stability. Statement (*iii*) resembles Proposition 11 (*iii*) but follows from a different dynamic pattern as the organization grows via $|T| \to \infty$ and via $\hat{f} \to \infty$, respectively. To illustrate, we compare g_{cp} and g_{cc}, both have in total $|T|(|T| - 1)/2$ links between managers. When $\hat{f}$ is fixed and $|T| \to \infty$, the number of links between a manager and a subordinate in g_{cp} is of order $|T|^2$ and linear in $\hat{f}$, while the number of links between a manager and a subordinate and between subordinates is of order $\hat{f}^2$ and linear in $|T|$. Meanwhile, both the transitive reductions of g_{cp} and g_{cc} are linear in both $|T|$ and $\hat{f}$. Thus, the relative density between g_{cp} and g_{cc} plays a major role in their stability comparison, and g_{cp} is more stable when $|T| \to \infty$ but less stable when $\hat{f} \to \infty$. A similar argument applies to the comparison between g_{cs} and g_{hc}.

Type-specific stability measure. Besides overall stability of a network, the above stability measure also allows for evaluating the stability of a certain type of links. Consider a network representing a flat organization, $g \in \{g_{dcp}, g_{cp}, g_{cc}, g_{cs}\}$. We may define a series of measures, $S_{mm}(g)$, $S_{ms}(g)$ and $S_{ss}(g)$ based on the type of link, where m stands for manager and s for subordinate; thus, the subscript mm refers to link between two managers, ms refers to link between a manager and a subordinate, and ss refers to link between two subordinates. The measure for each type of link is then calculated, analogous to the overall stability measure, by the largest possible fraction of such links that can be severed to preserve the network's transitive reduction. We list these measures in Table 2.

Network\Measure	$S_{mm}(g)$	$S_{ms}(g)$	$S_{ss}(g)$
Connected stars	$\frac{\lvert T\rvert-2}{\lvert T\rvert}$	0	$\varnothing$
Connected cliques	$\frac{\lvert T\rvert-2}{\lvert T\rvert}$	$\in (\frac{\hat{f}-2}{\hat{f}}, \frac{\hat{f}-1}{\hat{f}})$	$\in (\frac{\hat{f}-2}{\hat{f}}, \frac{\hat{f}-2}{\hat{f}} + \frac{2}{\hat{f}(\hat{f}-1)})$
Core-periphery	1	$\in (\frac{\lvert T\rvert-2}{\lvert T\rvert}, \frac{\lvert T\rvert-1}{\lvert T\rvert})$	$\varnothing$
Dense core-periphery	1	$\frac{(\hat{f}-1)\lvert T\rvert^2+(\lvert T\rvert-1)^2}{\hat{f}\lvert T\rvert^2}$	$\frac{\hat{f}-2}{\hat{f}}$

Table 2: Type-specific stability measures for flat networks.

Similarly, for a network representing a hierarchical organization, $g \in \{g_{hc}, g_{hs}\}$, we may consider type-specific measures $S_{tm}(g)$, $S_{ms}(g)$ and $S_{ss}(g)$. The subscript tm refers to link between the top manager and a team manager. We list these measures in Table 3.

Network\Measure	$S_{tm}(g)$	$S_{ms}(g)$	$S_{ss}(g)$
Hierarchical stars	0	0	$\varnothing$
Hierarchical cliques	0	$\frac{\hat{f}-1}{\hat{f}}$	$\frac{\hat{f}-2}{\hat{f}}$

Table 3: Type-specific stability measures for hierarchical networks.

The above findings give rise to several interesting patterns. Whenever a subgraph of the network displays a star structure, such as within a team in g_{cs} or between the top manager and team managers in a hierarchical network, the relevant links tend to be the least stable. This instability arises because each link is essential for the transitive reduction. Conversely, in clique structures like those within a team in g_{cc} or g_{hc}, the links are relatively stable, as most of them remain inactive in the transitive reduction. As an extreme case, full stability occurs for links between managers in g_{dcp} or g_{cp}, where none of such links is essential. This reveals the exclusive role of each manager in these networks, that is, to always serve as the intermediaries between two groups of subordinates. Moreover, since each subordinate is only directly linked to at most two managers in the transitive reduction of these networks, links between a manager and a subordinate are highly stable as well.

7 Conclusion

In this paper, we proposed a tractable framework to study an incentive design problem in team(s) where members have access to private internal information about each other's effort level. The feasible information architecture is described by an exogenous network, and the principal may exploit this architecture to minimize total rewards needed by endogenously determining the work sequence. Our results highlight two important leverages for the principal to reduce incentive cost: *improving* internal transparency of the action by every agent, and *balancing* internal transparency according to its diminishing marginal benefit. The optimal sequence exhibits distinct properties according to the network topology, e.g., monotone patterns in non-divisional networks and non-monotone patterns in divisional networks. We also establish explicit conditions under which the principal prefers global connections across teams, or local connections within teams.

Internal information in organizations remains an intriguing and promising topic in both theoretical and empirical economics, and related fields such as operations management and organization science. Besides hidden action as studied in this paper, the issue of internal transparency also interacts with other important economic forces about agents' private types, knowledge, evolution and updating, and so forth. For example, consider a scenario where hidden information and hidden action coexist: The agents in a team may face common uncertainty (a state of nature) about the project's success likelihood mapping, while each of them receives some private informative signal about the state. In this case, the observation of a peer's effort, or lack of effort, reveals information about the peer's signal and thus the state. As a perceivable contrast to our results in this paper, the principal would sometimes implement simultaneous moves, to avoid domino effects of low efforts caused by a possible unfavorable signal. We expect richer further studies to be conducted, with our work as part of the groundwork, on revealing relations between the nature of internal information and the optimal incentive design in a more general and flexible strategic environment.

A Appendix

A.1 Proofs

Proof of Lemma 1

Proof. Note that given the network g and sequence π, the set M_i is well-defined for each i. Thus, the proof follows directly from the proof of Winter (2010, Proposition 4). □

Proof of Lemma 2

Proof. Suppose by contradiction $\pi_i^* = \pi_j^*$. Consider a new sequence π' which only differs in that j moves immediately after i and before all the agents who move after i in π^*; thus, $\pi_j' > \pi_j^*$ and $\pi_k' = \pi_k^*$ for any agent $k \neq j$. It follows directly that $N_j^* \subset N_j'$ and $M_j' = M_j^*$. Consider an agent $k \neq j$. Clearly, if $\pi_k^* > \pi_j^*$, then $M_k' = M_k^*$. If $\pi_k^* \leq \pi_j^*$, then we partition M_k^* into two groups: $M_{k\backslash j}^*$ and $M_k^* \backslash M_{k\backslash j}^*$, where $M_{k\backslash j}^*$ is the set of agents who will remain in M_k^* if all j's links are eliminated and the agents move following π^*. Pick any agent $l \in M_k^*$. If $l \in M_{k\backslash j}^*$, then clearly he will remain in M_k'. If $l \in M_k^* \backslash M_{k\backslash j}^*$, it must be that $l \in M_j^*$. Since $N_j^* \subset N_j'$ and $M_j' = M_j^*$, l will still remain in M_k', and thus, $M_k^* \subseteq M_k'$. In summary, for any agent $k \in I$, $M_k^* \subseteq M_k'$, then by Lemma 1, $v_k^*(\pi') \leq v_k^*(\pi^*)$. Since $ij \in g$ and $\pi_i^* = \pi_j^*$, $M_i^* \subset M_i'$; thus, $v_i^*(\pi') < v_i^*(\pi^*)$. It follows that $V^*(\pi') < V^*(\pi^*)$, a contradiction. □

Proof of Lemma 3

Proof. Since either $i \in M_j^*$ or $j \in M_i^*$, both i's and j's neighbors are nonempty. There are thus two cases to consider. First, suppose $ij \notin g$, then there exists an agent $l \neq i, j$ such that $il, jl \in g$ and l moves between i and j. Suppose $\pi_i^* < \pi_l^* < \pi_j^*$, then $j, l \in M_i^*$. Now, swap i and j and denote the new sequence π'. Then, $i, l \in M_j'$. Since $\{k | ik \in g, k \neq j\} = \{k | jk \in g, k \neq i\}$, $N_i' = N_j^*$, $N_j' = N_i^*$, $M_i' = M_j^*$ and $M_j' \cup \{j\} = M_i^* \cup \{i\}$. Thus, for any agent $k \neq i, j$, there are three possibilities. First, $i, j \notin M_k^*$. Since $N_i' = N_j^*$ and $N_j' = N_i^*$, $M_k' = M_k^*$; thus, $v_k^*(\pi') = v_k^*(\pi^*)$. Second, $i \in M_k^*$, then $j \in M_k^*$ as $j \in M_i^*$. Since $M_i' = M_j^*$ and $M_j' \cup \{j\} = M_i^* \cup \{i\}$, $M_k' = M_k^*$; thus, $v_k^*(\pi') = v_k^*(\pi^*)$. Third, $j \in M_k^*$ and $i \notin M_k^*$, thus there exists an agent $k' \in M_k^*$ with $ik', jk' \in g$. Then by Lemma 2, we have $\pi_i^* < \pi_{k'}^* < \pi_j^*$. It follows that $M_k^* \backslash \{j\} = M_k' \backslash \{i\}$, and thus, we have

$$
\begin{aligned}
p(I \backslash (\{k\} \cup M_k^*)) &= p((I \backslash (\{k\} \cup (M_k^* \backslash \{j\})) \backslash \{j\}) = p((I \backslash (\{k\} \cup (M_k' \backslash \{i\})) \backslash \{j\}) \\
&> p((I \backslash (\{k\} \cup (M_k' \backslash \{i\})) \backslash \{i\}) = p(I \backslash (\{k\} \cup (M_k' \backslash \{i\}) \cup \{i\})) = p(I \backslash (\{k\} \cup M_k')).
\end{aligned}
$$

The inequality is because i is more important than j. Then, by Lemma 1, $v_k^*(\pi') < v_k^*(\pi^*)$. Moreover, since $M_i' = M_j^*$, we have

$$p(I\backslash(\{j\} \cup M_j^*)) = p((I\backslash M_j^*)\backslash\{j\}) > p((I\backslash M_j^*)\backslash\{i\}) = p(I\backslash(\{i\} \cup M_i')).$$

It follows from Lemma 1 that $v_i^*(\pi') < v_j^*(\pi^*)$. Lastly, since $M_j' \cup \{j\} = M_i^* \cup \{i\}$, we have $v_j^*(\pi') = v_i^*(\pi^*)$. Together, we have $V^*(\pi') < V^*(\pi^*)$, a contradiction. Thus, $\pi_j^* < \pi_l^* < \pi_i^*$.

Second, suppose $ij \in g$, then by Lemma 2, $\pi_i^* \neq \pi_j^*$, and thus, either $i \in N_j^*$ or $j \in N_i^*$. Suppose $\pi_i^* < \pi_j^*$. Again, swap i and j and denote the new sequence π'. Note that we now have $N_i' \cup \{i\} = N_j^* \cup \{j\}$, $N_j' = N_i^*$, $M_i' = M_j^*$ and $M_j' \cup \{j\} = M_i^* \cup \{i\}$. Analogously, the above argument goes through in this case, so $\pi_j^* < \pi_i^*$. In summary, if in π^* either $i \in M_j^*$ or $j \in M_i^*$, then $\pi_j^* < \pi_i^*$. Thus, the lemma is proven. □

Proof of Proposition 1

Proof. Since g is a clique network, by Lemma 2, the agents move sequentially in π^*. Also, note that for any two agents i and j, $\{k|ik \in g, k \neq j\} = \{k|jk \in g, k \neq i\}$. It follows from Lemma 3 that if i is more important than j, then $\pi_i^* > \pi_j^*$. By induction, we have that π^* is the identity permutation. Thus, the proposition is proven. □

Proof of Corollary 2

Proof. Let g_1 be a clique network and g_2 be an arbitrary network with the identical set of nodes as g_1. Let $\pi^*(g_2)$ be the optimal sequence for g_2. Consider a sequence $\pi(g_1)$ for g_1 such that each agent has the same order in $\pi(g_1)$ as in $\pi^*(g_2)$. By induction, one can easily show that for each agent i, $M_i(\pi^*(g_2)) \subseteq M_i(\pi(g_1))$. Then by Lemma 1, $v_i^*(\pi(g_1)) \leq v_i^*(\pi^*(g_2))$. It follows that $V^*(\pi^*(g_1)) \leq V^*(\pi(g_1)) \leq V^*(\pi^*(g_2))$ Thus, the corollary is proven. □

Proof of Lemma 5

Proof. By Corollary 1, in $MB(m)$, the value between brackets is decreasing in m, so is the difference between the last two terms. Moreover, the upper bound of summation $n-2-m$ is decreasing in m. Together, we have that $MB(m)$ is decreasing in m. On the other hand, by the definition of importance, $[p(I) - p(I\backslash\{n-m-1\})]^{-1}$ is increasing in m. In contrast, by the monotonicity of p, $[p(I) - p(\{j|j < n-m-1\})]^{-1}$ is decreasing in m. Together, we have that $MC(m)$ is increasing in m. Thus, the lemma is proven. □

Proof of Proposition 2

Proof. The optimal sequence π^* follows directly from Lemmas 4 and 5. It remains to prove the last two sentences of the proposition. First, suppose the agents are equally important to the project, then the input of $p(\cdot)$ reduces to the number or working agents. As a result, we can rewrite $V^*(m+1) - V^*(m)$ based on (2) as

$$\frac{p(n-1) - p(n-1-m)}{[p(n) - p(n-1)][p(n) - p(n-1-m)]} - \frac{(n-2-m)[p(n-2-m) - p(n-3-m)]}{[p(n) - p(n-2-m)][p(n) - p(n-3-m)]}$$

Clearly, if $m = 0$, then $V^*(m+1) - V^*(m) < 0$, meaning that $m^* \geq 1$. Moreover, since $p(\cdot)$ satisfies complementarity, we have that

$$p(n-1) - p(n-1-m) = \sum_{i=1}^{m} [p(n-i) - p(n-i-1)] > m[p(n-m) - p(n-1-m)].$$

This implies that $V^*(m+1) - V^*(m)$ is strictly greater than

$$\begin{aligned}
&\frac{m[p(n-m) - p(n-1-m)]}{[p(n) - p(n-1)][p(n) - p(n-1-m)]} - \frac{(n-2-m)[p(n-2-m) - p(n-3-m)]}{[p(n) - p(n-2-m)][p(n) - p(n-3-m)]} \\
&> \frac{m[p(n-m) - p(n-1-m)]}{[p(n) - p(n-1)][p(n) - p(n-1-m)]} - \frac{(n-2-m)[p(n-2-m) - p(n-3-m)]}{[p(n) - p(n-1)][p(n) - p(n-1-m)]} \\
&> \frac{(2m+2-n)[p(n-m) - p(n-1-m)]}{[p(n) - p(n-1)][p(n) - p(n-1-m)]},
\end{aligned}$$

where the first inequality is due to the monotonicity of p, and the second is again due to the complementarity of p. It follows that if $m \geq n/2 - 1$, then $V^*(m+1) - V^*(m) > 0$. This in turn implies that $m^* \leq n/2 - 1$. In summary, we have $1 \leq m^* < n/2$.

Finally, suppose $[p(I) - p(I \backslash \{n\})] \geq (n-1) [p(I) - p(I \backslash \{n-1\})]$; that is, the center is sufficiently more important than any other agent. Then, by direct calculation, we have

$$\begin{aligned}
V^*(1) - V^*(0) &> \frac{1}{p(I) - p(I \backslash \{n-1\})} - \sum_{i=1}^{n-2} \frac{1}{p(I) - p(\{j|j<n\} \backslash \{i\})} - \frac{1}{p(I) - p(\{j|j<n\})} \\
&> \frac{1}{p(I) - p(I \backslash \{n-1\})} - \frac{n-1}{p(I) - p(I \backslash \{n\})} \geq 0.
\end{aligned}$$

The first inequality follows from eliminating the positive term between brackets in (2), the second follows from the monotonicity of p, and the last is by the above assumption. It thus follows from Lemma 5 that $m^* = 0$. Thus, the proposition is proven. $\square$

Proof of Proposition 3

Proof. Since each manager is connected to all the other agents, by Lemma 2, in $\pi^*(g_{dcp})$, the managers move sequentially, and a manager and a subordinate cannot move simultaneously. Thus, the $|T|$ managers divide $\pi^*(g_{dcp})$ into $|T|+1$ "boxes"; each box can have none or some subordinates. Moreover, since two subordinates are directly linked only in the same team, if some box has multiple subordinates, then they must be from the same team. Suppose by contradiction there exist more than two adjacent managers in $\pi^*(g_{dcp})$, then due to the line structure of $\pi^*(g_{dcp})$, at least one manager is not adjacent to any subordinate in $\pi^*(g_{dcp})$. It follows that there are at most $|T|-1$ boxes that have subordinates. However, the previous argument implies that to allocate the $|T|$ teams of subordinates, we need at least $|T|$ boxes, a contradiction. Thus, the proposition is proven. □

Proof of Proposition 4

Proof. By a similar argument as in proving Proposition 3, the $|T|$ managers divide $\pi^*(g_{cp})$ into $|T|+1$ boxes; each box can have none or some subordinates. Suppose by contradiction there exist two adjacent managers l_t and $l_{t'}$ in $\pi^*(g_{cp})$, with l_t moving before $l_{t'}$, then there are at most $|T|$ boxes that have subordinates. Since there are at least $2|T|$ subordinates, at least one box has multiple subordinates; choose the box that is closest to l_t and $l_{t'}$, so that each box in between has at most one subordinate. If this box is before l_t and $l_{t'}$, then move one of the subordinates in the box, denoted i, to between l_t and $l_{t'}$; denote this sequence π'. Also, denote the manager who moves immediately after i in $\pi^*(g_{cp})$ $l_{t''}$. Note that in $\pi^*(g_{cp})$, i and all agents from $l_{t''}$ to l_t form a line, whereas in π', i moves from the front to the end of the line. It is easy to see that the total rewards to these agents are the same between $\pi^*(g_{cp})$ and π'. In addition, for any subordinate $j \neq i$ who moves immediately before $l_{t''}$ in $\pi^*(g_{cp})$, $|M'_j| = |M^*_j| + 1$, and for any other agent k, $|M'_k| = |M^*_k|$. Thus, it follows from Lemma 1 that $V^*(\pi') < V^*(\pi^*(g_{cp}))$, a contradiction. If this box is after l_t and $l_{t'}$, then again move a subordinate i in the box to between l_t and $l_{t'}$; denote this sequence π'. Also, denote the manager who moves immediately before i in $\pi^*(g_{cp})$ $l_{t''}$. Note that in $\pi^*(g_{cp})$, all agents from l_t to $l_{t''}$ together with i form a line. It is easy to show that the net change in transparency from $\pi^*(g_{cp})$ to π' is as if the transparency of i's action increased from $|M^*_i|$ to $|M^*_{l_{t''}}|$, while the transparency of any other action had not changed. It follows that $V^*(\pi') < V^*(\pi^*(g_{cp}))$, a contradiction. In summary, any two managers cannot be adjacent in $\pi^*(g_{cp})$. Then, by an analogous argument, we can easily show that any manager cannot move either the first or the last in the entire sequence under $\pi^*(g_{cp})$.

The above argument implies that managers and subordinates alternate in $\pi^*(g_{cp})$. Then, it remains to show that the number of subordinates between managers, s_q, is nonincreasing in q. Suppose not, then there is a manager l_t moving immediately after s_t subordinates and immediately before s'_t subordinates, such that $s'_t \geq s_t + 1$. Consider a new sequence π' such that one of the s'_t subordinates, denoted i, moves immediately before l_t, without changing any other relative order among agents. Thus, we have $|M'_i| = |M^*_i| + s'_t$. Moreover, for l_t and each of the s_t subordinates j, we have $|M'_{l_t}| = |M^*_{l_t}| - 1$ and $|M'_j| = |M^*_j| - 1$, and for any other agent k, we have $|M'_k| = |M^*_k|$. By Lemma 1, $v^*_i(\pi') - v^*_i(\pi^*) = [p(n) - p(n-1-|M^*_i| - s'_t)]^{-1} - [p(n) - p(n-1-|M^*_i|)]^{-1}$, $v^*_{l_t}(\pi') - v^*_{l_t}(\pi^*) = [p(n) - p(n-|M^*_{l_t}|)]^{-1} - [p(n) - p(n-1-|M^*_{l_t}|)]^{-1}$, and $v^*_j(\pi') - v^*_j(\pi^*) = [p(n) - p(n - |M^*_j|)]^{-1} - [p(n) - p(n-1-|M^*_j|)]^{-1}$ for each j. Note that $|M^*_{l_t}| < |M^*_j|$; thus, by Corollary 1, $v^*_{l_t}(\pi') - v^*_{l_t}(\pi^*) > v^*_j(\pi') - v^*_j(\pi^*)$. Then, by direct calculation and rearranging, the change in total rewards is given by

$$\begin{aligned}
V^*(\pi') - V^*(\pi^*) = &- \frac{p(n-1-|M^*_i|) - p(n-1-|M^*_i| - s'_t)}{[p(n) - p(n-1-|M^*_i|)][p(n) - p(n-1-|M^*_i| - s'_t)]} \\
&+ \frac{p(n-|M^*_{l_t}|) - p(n-1-|M^*_{l_t}|)}{[p(n) - p(n-|M^*_{l_t}|)][p(n) - p(n-1-|M^*_{l_t}|)]} \\
&+ \frac{s_t[p(n-|M^*_j|) - p(n-1-|M^*_j|)]}{[p(n) - p(n-|M^*_j|)][p(n) - p(n-1-|M^*_j|)]} \\
< &- \frac{s'_t[p(n-|M^*_i| - s'_t) - p(n-1-|M^*_i| - s'_t)]}{[p(n) - p(n-1-|M^*_i|)][p(n) - p(n-1-|M^*_i| - s'_t)]} \\
&+ \frac{(s_t+1)[p(n-|M^*_{l_t}|) - p(n-1-|M^*_{l_t}|)}{[p(n) - p(n-|M^*_{l_t}|)][p(n) - p(n-1-|M^*_{l_t}|)]} \\
< &\frac{(s_t+1-s'_t)[p(n-|M^*_{l_t}|) - p(n-1-|M^*_{l_t}|)}{[p(n) - p(n-|M^*_{l_t}|)][p(n) - p(n-1-|M^*_{l_t}|)]} \leq 0.
\end{aligned}$$

The first inequality is because by complementarity, $p(n - |M^*_i| - r) - p(n-1-|M^*_i| - r) > p(n - |M^*_i| - s'_t) - p(n-1-|M^*_i| - s'_t)$ for any positive integer $r < s'_t$, and $v^*_{l_t}(\pi') - v^*_{l_t}(\pi^*) > v^*_j(\pi') - v^*_j(\pi^*)$. The second inequality is due to monotonicity, and that $|M^*_{l_t}| = |M^*_i| + s'_t$, as managers and subordinates alternate in π^*. The last inequality is because $s'_t \geq s_t + 1$. This leads to a contradiction. Thus, the proposition is proven. □

Proof of Lemma 6

Proof. Statements (i), (iii) and (iv) follow simply from the argument in the text. It remains to prove statement (ii). Suppose in $\pi^*(g_{cc})$ there are two type-II teams t and t', and t moves before t'. Let m^*_t and $m^*_{t'}$ be the numbers of successors in t and t', respectively. Suppose by contradiction $m_t^* \leq m_{t'}^*$, then consider a new sequence π' in which $m_t' = m_t^* - 1$, $m_{t'}' = m_{t'}^* + 1$, and in each team agents form a line, without changing anything else. Let i be the immediate successor of l_t within t in $\pi^*(g_{cc})$, and j be the immediate successor of $l_{t'}$ within t' in π'. It is easy to show that the net change in action transparency between $\pi^*(g_{cc})$ and π' within t is as if i's transparency increased from $|M_i^*|$ to $|M_{l_t}'|$, and that within t' is as if $l_{t'}$'s transparency decreased from $|M_{l_{t'}}^*|$ to $|M_j'|$. In addition, the actions of all the managers and predecessors between l_t and $l_{t'}$ are more transparent in π' since $m_{t'}' = m_{t'}^* + 1$, while the transparency of any other action remains the same since $m_t' + m_{t'}' = m_t^* + m_{t'}^*$. Thus, it suffices to show that the decrease in rewards for t exceeds the increase in rewards for t'. By Lemma 1, the former value is given by $[p(n) - p(n-1-|M_i^*|)]^{-1} - [p(n) - p(n-1-|M_{l_t}'|)]^{-1}$, and the latter value is given by $[p(n) - p(n-1-|M_j'|)]^{-1} - [p(n) - p(n-1-|M_{l_{t'}}^*|)]^{-1}$. Note that $|M_i^*| = m_t^* - 1$ and $|M_j'| = m_{t'}^*$; thus, $|M_i^*| < |M_j'|$ as $m_t^* \leq m_{t'}^*$. Note too that $|M_{l_t}'| > |M_{l_{t'}}^*|$ since t moves before t' and $m_t' + m_{t'}' = m_t^* + m_{t'}^*$. It follows from the monotonicity of p that the decrease in rewards for t exceeds the increase in rewards for t', a contradiction. Thus, $m_t^* > m_{t'}^*$.

Then, fix the above t and t'. Suppose by contradiction there are fewer predecessors in t than in t', i.e., $f_t - m_t^* \leq f_{t'} - m_{t'}^*$, then switch the position of t and t', keeping the relative order among agents within each team; denote the new sequence π'. Let i be any predecessor in t, and j be the predecessor in t', who has the same distance to own manager as i. Since $m_t^* > m_{t'}^*$, $|M_i'| > |M_j^*|$ and $|M_j'| = |M_i^*|$. Thus, the total rewards to i and j are lower in π' than in $\pi^*(g_{cc})$. Note that the actions of all the managers and predecessors between t and t' are more transparent in π' as $m_t^* > m_{t'}^*$, so is the action of any remaining predecessor in t'. Note too that for any other agent k, $|M_k'| = |M_k^*|$. Together, we have $V^*(\pi') < V^*(\pi^*(g_{cc}))$, a contradiction. Thus, statement (ii) and therefore the proposition are proven. $\square$

Proof of Proposition 5

Proof. It remains to prove the last two sentences of the proposition. First, suppose for any two teams t and t', $1 + f_t \leq 2(1 + f_{t'})$. Choose two arbitrary teams t and t' under $\pi^*(g_{cc})$, such that t moves before t'. Suppose by contradiction t is a type-II team and t' is a type-I team. Let m_t be the number of successors in t. If $f_{t'} \geq m_t$, then consider a new sequence π' such that t becomes a type-I team, and t' has now $m_{t'} = m_t$ successors and thus $f_{t'} - m_{t'} \geq 0$ predecessors, without changing anything else. It is easy to see that such a change essentially moves m_t successors from t to t', and m_t predecessors from t' to t. Since t moves before t', such a change clearly improves transparency; thus, $V^*(\pi') < V^*(\pi^*(g_{cc}))$, a contradiction. If $f_{t'} < m_t$, then $f_{t'} \geq f_t - m_t$ since $1 + f_t \leq 2(1 + f_{t'})$. Switch the position of t and t', keeping the relative order among agents within each team; denote the new sequence π'. By the same argument as in the last paragraph in the proof of Lemma 6, we have $V^*(\pi') < V^*(\pi^*(g_{cc}))$, a contradiction. In summary, a type-II team must move after any type-I team, and before any type-III team. In particular, suppose team size is identical across teams, then there can be at most one type-II team in $\pi^*(g_{cc})$. Suppose by contradiction there are multiple type-II teams, then by the above, there are two adjacent type-II teams. But, by Lemma 6, we have that both the numbers of predecessors and successors in the earlier type-II team are strictly less than those of the later type-II team. This contradicts that each team has the same size. Thus, the proposition is proven. □

Proof of Corollary 3

Proof. Consider the algorithm in the corollary. Suppose step 2 now reaches team t', such that there are $m_{t'}$ successors in t'. There are two cases. First, if $m_{t'} < f_{t'}$, then $MB(\sum m_t)$ equals the reduction in rewards for all agents in teams before t'. It follows from Corollary 1 that $MB(\sum m_t)$ decreases in $m_{t'}$ for fixed t' and $m_{t'} < f_{t'}$. On the other hand, $MC(\sum m_t)$ equals the increase in rewards for agents in t', as if the manager $l_{t'}$'s action could be learned by only $m_{t'}$, instead of $|M_{l_{t'}}|$, agents. Thus, by Lemma 1, we have

$$MC(\sum m_t) = \frac{1}{p(n) - p(n-1-m_{t'})} - \frac{1}{p(n) - p(n-1-|M_{l_{t'}}|)}.$$

If t' is the last team in the sequence, then $|M_{l_{t'}}| = m_{t'}$, and thus, $MC(\sum m_t) = 0$; otherwise, $|M_{l_{t'}}| - m_{t'}$ is a positive constant given that $m_{t'} < f_{t'}$, which equals the number of agents in all teams after t'. By Corollary 1, $MC(\sum m_t)$ decreases in $m_{t'}$ for fixed t' and $m_{t'} < f_{t'}$.

Second, if $m_{t'} = f_{t'}$, then t' is not the first team in the sequence, since in $\pi^*(g_{cc})$ the first team is type-I. Let t'' be the team immediately before t'. In this case, $MB(\sum m_t)$ equals the reduction in rewards for all agents in teams before t'', indicating that $MB(\sum m_t)$ decreases across teams. Combining the above, we have that $MB(\sum m_t)$ always decreases in $\sum m_t$. In contrast, $MC(\sum m_t)$ equals the increase in rewards for agents in t'', as if the manager $l_{t''}$'s action could be learned by 0, instead of $|M_{l_{t''}}|$, agents. Since $m_{t'} \geq 0$ and $|M_{l_{t''}}| > |M_{l_{t'}}|$ for any $\sum m_t$, by the above equation, $MC(\sum m_t)$ increases in $\sum m_t$ across teams, but decreases in $\sum m_t$ within a team. Moreover, for any $m_{t'} = m_{t''} < f_{t'} = f_{t''}$, the increase in rewards for t' is lower than that for t'', as $|M_{l_{t''}}| > |M_{l_{t'}}|$. This implies that $MB(\sum m_t)$ and $MC(\sum m_t)$ can intersect in at most one team; before reaching that team, $MB(\sum m_t) > MC(\sum m_t)$ for all $\sum m_t$, and afterward, the opposite is true. Therefore, the algorithm is valid. □

Proof of Proposition 6

Proof. We first show that in $\pi^*(g_{cs})$, any predecessor i must move before any successor j. We only need to consider when i and j are from two different teams t and t', respectively. Suppose by contradiction t moves after t', then make i a successor and j a predecessor; let π' denote the new sequence. Consider an arbitrary agent $k \neq i, j$. There are three possibilities. If $i \notin M_k^*$ and $j \in M_k^*$, then $|M_k'| = |M_k^*|$. If $i, j \notin M_k^* \cup M_k'$, then $|M_k'| = |M_k^*|$. If $i, j \notin M_k^*$, $i \in M_k'$, and $j \notin M_k'$, then $|M_k'| > |M_k^*|$. It follows from Lemma 1 that $v_k^*(\pi') \leq v_k^*(\pi^*(g_{cs}))$, with strict inequality in some cases. Moreover, note that $|M_i'| = |M_j^*| = 0$, and $|M_i^*| < |M_j'|$ since t moves after t'. Thus, $v_i^*(\pi') + v_j^*(\pi') < v_i^*(\pi^*(g_{cs})) + v_j^*(\pi^*(g_{cs}))$. In summary, we have that $V^*(\pi') < V^*(\pi^*(g_{cs}))$, a contradiction. This implies that in $\pi^*(g_{cs})$, there is at most one type-II team, which moves after all type-I teams if they exist, and before all type-III teams if they exist. If a type-II team does not exist, then $\pi^*(g_{cs})$ is such that a set of type-I teams is succeeded by a set of type-III teams. Moreover, similar to a single star, it is easy to show that in $\pi^*(g_{cs})$, the subordinates cannot be all predecessors or all successors. Lastly, similar to a connected-cliques network, it is easy to show that in $\pi^*(g_{cs})$, type-I (type-III) teams are organized in descending (ascending) order of size. Thus, the proposition is proven. □

Proof of Corollary 4

Proof. Consider the algorithm in the corollary. From Corollary 1, we have that $MB(\sum m_t)$ is decreasing in $\sum m_t$. On the other hand, $MC(\sum m_t)$ is given by the increase in reward for the new successor who was initially a predecessor. Since a successor's action is unobservable, and the algorithm proceeds backward, by Lemma 1, $MC(\sum m_t)$ is increasing in $\sum m_t$. Thus, the algorithm is valid to completely characterize $\pi^*(g_{cs})$. □

Proof of Proposition 7

Proof. Suppose the managers are ordered according to $\pi^*(g_{cs})$, and consider the algorithm in Corollary 4. Fix $\sum m_t < \sum f_t$, let t be the last team in the sequence such that $m_t < f_t$. Note that any team before t is type-I, and any team after t is type-III. Thus, for any agent $i \neq l_t$, either $i \in M_{l_t}$ or $l_t \in M_i$. Then, reconfigure g_{cs} hypothetically into a star network in which l_t is the center and i is l_t's predecessor (successor) if $l_t \in M_i$ ($i \in M_{l_t}$). Note too that if in g_{cs}, $l_t \in M_i$, then for fixed $|M_{l_t}|$, i's action is (weakly) more transparent in g_{cs} than in the corresponding star. By Corollary 1, the marginal reduction in reward for i is (weakly) lower in g_{cs} than in the star, as i's action becomes more transparent. It follows that $MB(\sum m_t)$

in g_{cs} is lower than $MB(m)$ in the star if $\sum m_t = m$, since $|M_{l_t}| \geq \sum m_t$ in g_{cs}. In contrast, for fixed $|M_{l_t}|$, the increase in reward for the new successor is the same between g_{cs} and the star, since in both networks, the transparency of his action decreases from $|M_{l_t}|+1$ to 0. It follows that $MC(\sum m_t)$ in g_{cs} is higher than $MC(m)$ in the star if $\sum m_t = m$. In summary, we have that in $\pi^*(g_{cs})$, the total number of successors $\sum m_t^*$ is lower than the total number of successors m^* in the optimal sequence of the star. It suffices to show that m^* is bounded above by some number of order $\sqrt{n}$. From Section 4.2, $MB(m) - MC(m)$ is given by

$$\begin{aligned}
&\frac{n-m-2}{p(n)-p(n-m-2)} - \frac{n-m-2}{p(n)-p(n-m-3)} + \frac{1}{p(n)-p(n-m-1)} - \frac{1}{p(n)-p(n-m-2)} \\
&- \left[\frac{1}{p(n)-p(n-1)} - \frac{1}{p(n)-p(n-m-2)}\right] \\
&< \frac{(n-m-1)[p(n-m-1)-p(n-m-2)]}{[p(n)-p(n-m-1)][p(n)-p(n-m-2)]} - \frac{p(n-1)-p(n-m-2)}{[p(n)-p(n-1)][p(n)-p(n-m-2)]} \\
&< \frac{(n-m-1)[p(n-m-1)-p(n-m-2)]}{[p(n)-p(n-m-1)][p(n)-p(n-m-2)]} - \frac{(m+1)[p(n-m-1)-p(n-m-2)]}{[p(n)-p(n-1)][p(n)-p(n-m-2)]} \\
&\propto \frac{n-m-1}{p(n)-p(n-m-1)} - \frac{m+1}{p(n)-p(n-1)} \leq \frac{n-m-1}{p(n)-p(n-m-1)} - \frac{Km(m+1)}{p(n)-p(n-m)} \\
&< \frac{-Km^2-(K+1)m+n-1}{p(n)-p(n-m)}.
\end{aligned}$$

The first two inequalities are due to the monotonicity and complementarity of p. The third inequality is due to Assumption 1, and the last one is due to the monotonicity of p. Clearly, the larger root of $-Km^2-(K+1)m+n-1$ is of order $\sqrt{n}$, as $n \to \infty$, and for any integer m greater than that root, the function value is negative. Thus, $MB(m) - MC(m) < 0$ for m of higher order than $\sqrt{n}$. By (3), m^* is bounded above by some integer of order $\sqrt{n}$.

Then, we turn to a connected-cliques network g_{cc}. Let t be the last team in $\pi^*(g_{cc})$ such that any team before t, if it exists, is a type-I team. Let $m = \sum m_t^*$ be the total number of successors in $\pi^*(g_{cc})$. In Section 5.3, we showed that $MB(\sum m_t^*)$ is given by the reduction in rewards for all the managers and predecessors in teams before t, and $MC(\sum m_t^*)$ equals the increase in rewards for t, as if the transparency of l_t's action decreased from $|M_{l_t}^*|$ to m_t^*. Similarly, reconfigure g_{cc} into a star network in which l_t is the center, all the agents in teams before t are the predecessors of l_t, and all the other agents are the successors of l_t. Note that in this star, the number of predecessors of l_t is less than $n-m-1$, and each predecessor's action is (weakly) less transparent than in $\pi^*(g_{cc})$. It follows that $MB(\sum m_t^*)$ is lower than $MB(m)$ in the corresponding star. On the other hand, since $|M_{l_t}^*| \geq \sum m_t^* = m$, $MC(\sum m_t^*)$ is greater than it would be if the transparency of l_t's action decreased from $\sum m_t^*$ to m_t^*. In summary, we have that $MB(\sum m_t^*) - MC(\sum m_t^*)$ is less than

$$
\begin{aligned}
&\frac{n-m-2}{p(n)-p(n-m-2)} - \frac{n-m-2}{p(n)-p(n-m-3)} + \frac{1}{p(n)-p(n-m-1)} - \frac{1}{p(n)-p(n-m-2)} \\
&- \left[\frac{1}{p(n)-p(n-m_t^*)} - \frac{1}{p(n)-p(n-m-1)}\right] \\
&< \frac{(n-m-1)[p(n-m-1)-p(n-m-2)]}{[p(n)-p(n-m-1)][p(n)-p(n-m-2)]} - \frac{(m-m_t^*+1)[p(n-m-1)-p(n-m-2)]}{[p(n)-p(n-m_t^*)][p(n)-p(n-m-1)]} \\
&\propto \frac{n-m-1}{p(n)-p(n-m-2)} - \frac{m-m_t^*+1}{p(n)-p(n-m_t^*)} < \frac{n-m-1}{p(n)-p(n-m-2)} - \frac{m-m_t^*+1}{m_t^*[p(n)-p(n-1)]} \\
&\leq \frac{n-m-1}{p(n)-p(n-m-2)} - \frac{Km(m-m_t^*+1)}{m_t^*[p(n)-p(n-m)]} \\
&< \frac{-Km^2-(K+m_t^*-Km_t^*)m+m_t^*(n-1)}{m_t^*[p(n)-p(n-m)]}.
\end{aligned}
$$

The first inequality is again due to the monotonicity and complementarity of p. The second inequality is due to the complementarity of p. The third inequality is due to Assumption 1, and the last one is due to the monotonicity of p. Suppose as $n \to \infty$, f_τ is bounded for each team τ. Since $m_t^* < f_t$, m_t^* is also bounded. Thus, the larger root of the numerator in the last line above is also of order $\sqrt{n}$, as $n \to \infty$. It follows that $MB(m) - MC(m) < 0$ for m of higher order than $\sqrt{n}$. In turn, this implies that in g_{cc}, $\sum m_t^*$ must be bounded above by some integer of order $\sqrt{n}$, since otherwise, it is profitable to reduce $\sum m_t$, a contradiction. Thus, the proposition is proven. □

Proof of Proposition 8

Proof. We first show that in $\pi^*(g_{hc})$, any team before h is type-I. Suppose there exists a team t before h, which is not type-I, then make t a type-I team. Clearly, this will make all actions in t more transparent, with some strictly more transparent, and any actions outside t equally transparent, a contradiction. Next, suppose there exists a team t after h, which is not type-III, then make t a type-III team. Clearly, this will strictly improve the transparency of the actions of h and each agent i such that $h \in M_i^*$, and at least weakly improve that of each action in t, without changing the transparency of any other action, a contradiction. It follows that any team after h is type-III. Now, suppose no teams move before h, then choose a team t after h, move it to before h and make it a type-I team. Note that h and the agents in t form a line both before and after the adjustment. Note too that after the adjustment, the most transparent action within the line is equally transparent as before, and any action ranked lower in terms of transparency is now more transparent than its counterpart before the adjustment. Moreover, any action outside the line remains equally transparent after the adjustment, a contradiction. Thus, there must be a team before h. Next, suppose no teams move after h, then choose a team t before h, move it to after h and make it a type-III team. Similarly, h and the agents in t form a line both before and after the adjustment. It is easy to see that the total rewards within the line remains the same after the adjustment, whereas the reward for any other agent is lower, since his action becomes more transparent after the adjustment, a contradiction. Thus, there must be a team after h. Then, let x and y be the numbers of agents before and after h in $\pi^*(g_{hc})$, respectively, and let t be the smallest team after h. If $y \leq 2f_t$, then clearly $x > y - 2f_t$ since $x > 0$. Suppose $x \leq y - 2f_t$, then move t to before h and make it a type-I team. This change has two implications. On the one hand, the agents in t require less rewards since their actions become more transparent. On the other hand, those x agents require more rewards for the opposite reason. By the monotonicity of p and Corollary 1, the least reduction in reward in t is for the manager of t, who moves from immediately after h to immediately before h, per the above results; so the total reduction is more than $(1 + f_t)$ times the reduction for the manager. On the other hand, by Corollary 1, the largest increase in reward among those x agents is for a manager, since his action is the least transparent among the x agents' actions; so the total increase in rewards is less than x times the increase in reward for the manager. Together, the net reduction in total rewards due to the adjustment is greater than

$$
\begin{aligned}
&(1+f_t)\left[\frac{1}{p(n)-p(n-1-f_t)}-\frac{1}{p(n)-p(n-1-y+f_t)}\right] \\
&-x\left[\frac{1}{p(n)-p(n-1-y+f_t)}-\frac{1}{p(n)-p(n-2-y)}\right] \\
&>\frac{(1+f_t)(y-2f_t)[p(n-y+f_t)-p(n-1-y+f_t)]}{[p(n)-p(n-1-f_t)][p(n)-p(n-1-y+f_t)]} \\
&-\frac{x(1+f_t)[p(n-1-y+f_t)-p(n-2-y+f_t)]}{[p(n)-p(n-1-y+f_t)][p(n)-p(n-2-y)]} \\
&>\frac{(1+f_t)(y-2f_t-x)[p(n-y+f_t)-p(n-1-y+f_t)]}{[p(n)-p(n-1-f_t)][p(n)-p(n-1-y+f_t)]} \\
&\geq 0.
\end{aligned}
$$

The first inequality follows from the complementarity of p, the second one follows from the monotonicity and complementarity of p, and that $y > f_t - 1$, and the last one follows from the assumption that $x \leq y - 2f_t$. Therefore, the adjustment leads to a profitable deviation, a contradiction. Thus, the proposition is proven. □

Proof of Proposition 9

Proof. First, by a similar argument as in the proof of Proposition 8, it is easy to show that in $\pi^*(g_{hs})$, any type-I (type-III) team must move before (after) h, and h can be neither the first nor the last in $\pi^*(g_{hs})$. Suppose there exist two type-II teams separated by h, then it is clearly profitable to decrease one successor in the team before h and increase one successor in the team after h, a contradiction. That is, all type-II teams must be either before or after h. Suppose there exist two type-II teams, t' and t'', such that $f_{t'} < f_{t''}$. Suppose $m^*_{t'} > m^*_{t''}$, then $f_{t'} - m^*_{t'} < f_{t''} - m^*_{t''} - 1$. Since both t' and t'' are either before or after h, decreasing a successor within t' and increasing a successor within t'' will have no impact on the reward to any agent outside t' and t''. However, doing so reduces the total rewards to agents in t' and t''. To see this, let i and j be the adjusted agents in t' and t'', respectively. Note that the reduction in i's reward is (weakly) larger than the increase in j's reward, as $m^*_{t'} > m^*_{t''}$. Moreover, by Corollary 1, the increase in reward for $l_{t'}$ and each predecessor other than i in t' is (weakly) less than the decrease in reward for $l_{t''}$ and each remaining predecessor in t'', respectively, as $m^*_{t'} > m^*_{t''}$. Since $f_{t'} - m^*_{t'} < f_{t''} - m^*_{t''} - 1$, the total increase in rewards for all predecessors other than i in t' is less than the total decrease in rewards for all remaining predecessors in t''. Together, we have that the adjustment is profitable, a contradiction. Let x be the number of managers and predecessors before h, and y be the number of managers and successors after h, and let t be the smallest team after h, with m^*_t successors. If $y \leq 2f_t$, then $x > y - 2f_t$ since $x > 0$. Suppose $x \leq y - 2f_t$, then move t to before h and make it a type-I team. Analogous to the proof of Proposition 8, the least reduction in reward in t is for an agent who was initially a predecessor, as his action was the most transparent in t; the largest increase in reward among those x agents is again for a manager. Similarly, we have that the net reduction in total rewards due to the adjustment is greater than

$$\begin{aligned}
&(1+f_t)\left[\frac{1}{p(n)-p(n-2-m^*_t)} - \frac{1}{p(n)-p(n-2-y+m^*_t)}\right] \\
&- x\left[\frac{1}{p(n)-p(n-1-y+m^*_t)} - \frac{1}{p(n)-p(n-2-y)}\right] \\
&> \frac{(1+f_t)(y-2m^*_t-x)[p(n-1-y+m^*_t)-p(n-2-y+m^*_t)]}{[p(n)-p(n-2-m^*_t)][p(n)-p(n-2-y+m^*_t)]}.
\end{aligned}$$

The inequality is due to a similar argument as in the proof of Proposition 8. Since $m^*_t \leq f_t$, if $x \leq y - 2f_t$, then the term in the last line is nonnegative. This means that the adjustment leads to a profitable deviation, a contradiction. Thus, the proposition is proven. ∎

Proof of Proposition 10

Proof. We first consider a hierarchical-cliques network. Recall the adjustment in the proof of Proposition 8. We have shown that the net reduction in total rewards is greater than

$$\begin{aligned}&(1+f_t)\left[\frac{1}{p(n)-p(n-1-f_t)}-\frac{1}{p(n)-p(n-1-y+f_t)}\right]\\&-x\left[\frac{1}{p(n)-p(n-1-y+f_t)}-\frac{1}{p(n)-p(n-2-y)}\right],\end{aligned} \tag{4}$$

where x and y are the numbers of agents before and after h in $\pi^*(g_{hc})$, respectively, and t is the smallest team after h. By Proposition 8, we have that y equals the number of managers and successors after h, and thus, $x = n - 1 - y$. By the complementarity of p, we have

$$p(n)-p(n-1-f_t)=\sum_{i=1}^{1+f_t}[p(n+1-i)-p(n-i)]<(1+f_t)[p(n)-p(n-1)].$$

It follows that $p(n-1-f_t) > p(n)-(1+f_t)[p(n)-p(n-1)]$. Similarly, we have

$$\begin{aligned}&p(n-1-y+f_t)-p(n-y)<(f_t-1)[p(n)-p(n-1)] \text{ and}\\&p(n-1-y+f_t)-p(n-2-y)<(1+f_t)[p(n)-p(n-1)].\end{aligned}$$

It follows that $p(n-1-y+f_t) < p(n-y)+(f_t-1)[p(n)-p(n-1)]$. Substituting these results into (4), we have that (4) is greater than

$$\begin{aligned}&(1+f_t)\frac{p(n)-p(n-y)-2f_t[p(n)-p(n-1)]}{[p(n)-p(n-1-f_t)][p(n)-p(n-1-y+f_t)]}\\&-(n-1-y)\frac{(1+f_t)[p(n)-p(n-1)]}{[p(n)-p(n-1-y+f_t)][p(n)-p(n-2-y)]}\\&\propto\frac{p(n)-p(n-y)-2f_t[p(n)-p(n-1)]}{p(n)-p(n-1-f_t)}-\frac{(n-1-y)[p(n)-p(n-1)]}{p(n)-p(n-2-y)}\\&\geq\frac{(Ky-2f_t)[p(n)-p(n-1)]}{p(n)-p(n-1-f_t)}-\frac{(n-1-y)[p(n)-p(n-1)]}{K(y+2)[p(n)-p(n-1)]}\end{aligned} \tag{5}$$

The inequality is due to Assumption 1. If $Ky > 2f_t$, then (5) is greater than

$$\begin{aligned}&\frac{(Ky-2f_t)[p(n)-p(n-1)]}{(1+f_t)[p(n)-p(n-1)]}-\frac{(n-1-y)[p(n)-p(n-1)]}{K(y+2)[p(n)-p(n-1)]}\\&\propto K^2y^2+(2K^2-2Kf_t+1+f_t)y-4Kf_t-(1+f_t)(n-1).\end{aligned}$$

This is because $p(n) - p(n-1-f_t) < (1+f_t)[p(n) - p(n-1)]$. Since f_t is bounded by some constant, the larger root of the last line above is of order $\sqrt{n}$, as $n \to \infty$. Thus, for any y of higher order, (5) and thus (4) is strictly positive, meaning that the adjustment is profitable, a contradiction. If $Ky \leq 2f_t$, then y is bounded by a constant. Together, we have that y is bounded by some number of order $\sqrt{n}$, as $n \to \infty$.

Then, we turn to a hierarchical-stars network, and recall the adjustment in the proof of Proposition 9. We have shown that the net reduction in total rewards is greater than

$$\begin{aligned} &(1+f_t)\left[\frac{1}{p(n) - p(n-2-m_t^*)} - \frac{1}{p(n) - p(n-2-y+m_t^*)}\right] \\ &- x\left[\frac{1}{p(n) - p(n-1-y+m_t^*)} - \frac{1}{p(n) - p(n-2-y)}\right], \end{aligned} \tag{6}$$

where x is the number of managers and predecessors before h, and y is that of managers and successors after h, and t is the smallest team after h, with m_t^* successors. By Proposition 9, we have $x \leq n-1-y$ and $m_t^* \geq 1$. Then, by the monotonicity of p, (6) is greater than

$$\begin{aligned} &(1+f_t)\frac{p(n-2-m_t^*) - p(n-1-y+m_t^*)}{[p(n) - p(n-2-m_t^*)][p(n) - p(n-1-y+m_t^*)]} \\ &- (n-1-y)\frac{p(n-1-y+m_t^*) - p(n-2-y)}{[p(n) - p(n-1-y+m_t^*)][p(n) - p(n-2-y)]} \\ &> (1+f_t)\frac{[Ky - (1+2m_t^*)][p(n) - p(n-1)]}{[p(n) - p(n-2-m_t^*)][p(n) - p(n-1-y+m_t^*)]} \\ &- (n-1-y)\frac{(1+m_t^*)[p(n) - p(n-1)]}{[p(n) - p(n-1-y+m_t^*)][p(n) - p(n-2-y)]} \\ &\propto \frac{(1+f_t)[Ky - (1+2m_t^*)]}{p(n) - p(n-2-m_t^*)} - \frac{(1+m_t^*)(n-1-y)}{p(n) - p(n-2-y)} \\ &\geq \frac{(1+f_t)[Ky - (1+2m_t^*)]}{p(n) - p(n-2-m_t^*)} - \frac{(1+m_t^*)(n-1-y)}{K(y+2)[p(n) - p(n-1)]} \\ &\geq \frac{(1+f_t)[Ky - (1+2m_t^*)]}{p(n) - p(n-2-m_t^*)} - \frac{(1+f_t)(n-1-y)}{K(y+2)[p(n) - p(n-1)]} \\ &\propto \frac{Ky - (1+2m_t^*)}{p(n) - p(n-2-m_t^*)} - \frac{n-1-y}{K(y+2)[p(n) - p(n-1)]}. \end{aligned}$$

The first inequality follows from a similar argument as in hierarchical-cliques networks. The second inequality is due to Assumption 1, and the last one is because $m_t^* \leq f_t$. Then, by an analogous argument as in the above, we can show that (6) is strictly positive if y is of higher order than $\sqrt{n}$, as $n \to \infty$, leading to a contradiction. Thus, the proposition is proven. □

Proof of Proposition 11

Proof. We first prove statements (i) and (ii). Suppose c is sufficiently small such that $\tilde{g}$ is connected for each $g \in G$. Proposition 3 indicates that $\pi^*(g_{dcp})$ yields the most transparent and minimally connected monitoring network; thus, it is easy to see that $C(g_{dcp}) < C(g)$ for all $g \in G, g \neq g_{dcp}$. Also, Propositions 5 and 6 indicate that both $\pi^*(g_{cc})$ and $\pi^*(g_{cs})$ yield minimally connected monitoring networks for g_{cc} and g_{cs}, respectively. But, the monitoring network of g_{cc} is clearly more transparent than that of g_{cs}. It follows that $C(g_{cc}) < C(g_{cs})$. Regarding g_{cp} and g_{cs}, note that for any minimally connected monitoring network of g_{cs}, g_{cp} can yield the same minimally connected monitoring network, except that one subordinate is positioned between two managers (see Figure 8), leading to more internal information. This implies that $C(g_{cp}) < C(g_{cs})$. Now, consider g_{cc} and g_{hc}. Proposition 8 indicates that $\pi^*(g_{hc})$ yields a minimally connected monitoring network for g_{hc}.[7] Note that under both $\pi^*(g_{cc})$ and $\pi^*(g_{hc})$, agents form a line in each team. However, the managers form a line in $\pi^*(g_{cc})$, but are centering around the top manager in $\pi^*(g_{hc})$. It is easy to see that $\pi^*(g_{cc})$ leads to more internal information than $\pi^*(g_{hc})$; thus, $C(g_{cc}) < C(g_{hc})$. Similarly, we have $C(g_{hc}) < C(g_{hs})$ and $C(g_{cs}) < C(g_{hs})$. Together, we have that $C(g_{hs}) > C(g)$ for all $g \in G, g \neq g_{hs}$.

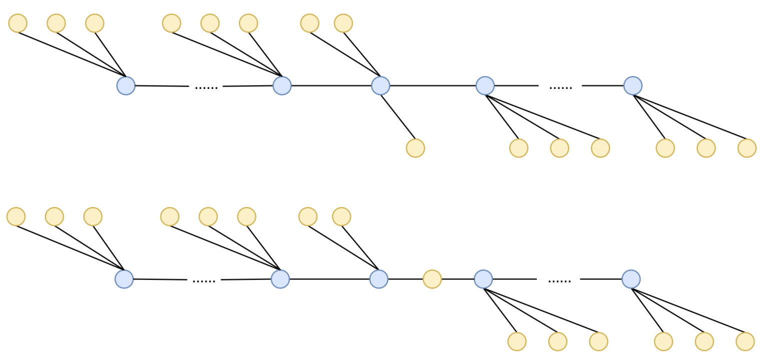


Figure 8: A minimally connected monitoring network of g_{cs} (top) and that of g_{cp} (bottom).

Then, we turn to statement (iii). First, fix $\hat{f}$ and let $|T| \to \infty$. Let V^*_{cp} and V^*_{cc} denote the minimized total rewards under the efficient sequences of g_{cp} and g_{cc}, respectively. Since $\pi^*(g_{cc})$ yields a minimally connected monitoring network, it is indeed an efficient sequence of g_{cc}. Since $f_t \equiv \hat{f}$, by Corollary 3, $\pi^*(g_{cc})$ can be characterized by a tuple, $(\overline{T}_{cc}, m^*_{t^*}, \underline{T}_{cc})$, where $m^*_{t^*}$ is the number of successors in team t^* characterized by Corollary 3, and $\overline{T}_{cc}$ $(\underline{T}_{cc})$ is the number of type-I teams (type-III teams) before (after) team t^*; clearly, these numbers depend on $|T|$. The lemma below shows that $\underline{T}_{cc}$ is of lower order of $|T|$, as $|T| \to \infty$.

[7] Given our assumption in footnote 6, for both g_{hc} and g_{hc}, a minimally connected monitoring network has n links, whereas that of any other $g \in G$ has $n-1$ links.

Lemma 7. *Given $\hat{f}$, $\lim_{|T|\to\infty} \underline{T}_{cc}/|T| = 0$.*

Proof. Note that along $\pi^*(g_{cc})$, it is successively $\overline{T}_{cc}$ type-I teams, team t^*, and $\underline{T}_{cc}$ type-III teams. Now, construct a similar sequence π', such that the managers form a line, and there is a team t' in an interior stage such that any team before t' is a type-I team and any team after t' is a type-III team. Let $|M'_{l_{t'}}|(n)$ be the action transparency of the manager of t', $l_{t'}$, when the population is n. Given $\hat{f}$, $n \to \infty$ as $|T| \to \infty$. Note that we can always construct such π' that as $n \to \infty$, $|M'_{l_{t'}}|(n) \to \infty$ and $|M'_{l_{t'}}|(n)/n \to 0$. Thus, given our construction of p in this section, $p(|M'_{l_{t'}}|(n)) \to p(0)$ as $n \to \infty$. Note too that for any agent $i \in M'_{l_{t'}}$, his reward is lower than $[p(n) - p(n-1)]^{-1}$, while for any agent j such that $l_{t'} \in M'_j$, his reward is lower than that of $l_{t'}$, which is $[p(n) - p(n-1-|M'_{l_{t'}}|(n))]^{-1}$. Since π' is not necessarily the optimal, the average reward under π' must be higher than under $\pi^*(g_{cc})$. Thus,

$$\limsup_{n\to\infty} \frac{V^*_{cc}}{n} \le \lim_{n\to\infty} \frac{1}{n}\left[\frac{|M'_{l_{t'}}|(n)}{p(n)-p(n-1)} + \frac{n-|M'_{l_{t'}}|(n)}{p(n)-p(n-1-|M'_{l_{t'}}|(n))}\right] = \frac{1}{p(n)-p(0)}.$$

On the other hand, note that any agent's reward is bounded below by $[p(n) - p(0)]^{-1}$; thus, we have $\lim_{n\to\infty} V^*_{cc}/n = [p(n)-p(0)]^{-1}$. However, for fixed $\hat{f}$, any subordinate's reward in a type-III team is significantly higher than $[p(n)-p(0)]^{-1}$. This implies that there can only be an infinitesimal fraction of type-III teams, as $|T| \to \infty$. That is, $\lim_{|T|\to\infty} \underline{T}_{cc}/|T| = 0$. □

Then, we have the following lemma.

Lemma 8. *$C(g_{cp}) < C(g_{cc})$ when $\hat{f}$ is fixed and $|T|$ is sufficiently large.*

Proof. For any sufficiently large $|T|$, we shall construct a sequence π' in g_{cp}, which yields a minimally connected monitoring network, and leads to strictly lower total rewards than V^*_{cc}. Suppose in $\pi^*(g_{cc})$, there exist T' teams with at least one successor in each team. Let n' be the total number of these successors; thus, $n' \le T'\hat{f}$. Then, in π', form a line of length $2|T|$ by alternating the managers and subordinates one by one, and then move all the remaining subordinates to before the first manager, as illustrated in Figure 9.

By Lemma 7, when $|T|$ is sufficiently large, $2|T| \gg T' + n'$. Since the last $T' + n'$ agents in π' form a line, their total rewards are strictly lower than those of the last $T' + n'$ agents in $\pi^*(g_{cc})$. On the other hand, among the first $|T| - T'$ teams in $\pi^*(g_{cc})$, the longest line has a length $\hat{f} + |T| - T'$. By Lemma 7, $2|T| - T' - n' \gg \hat{f} + |T| - T'$, for sufficiently large $|T|$. This means that the action of each agent before the first manager in π' is significantly more transparent than the most transparent action in $\pi^*(g_{cc})$. Since the remaining $2|T| - T' - n'$

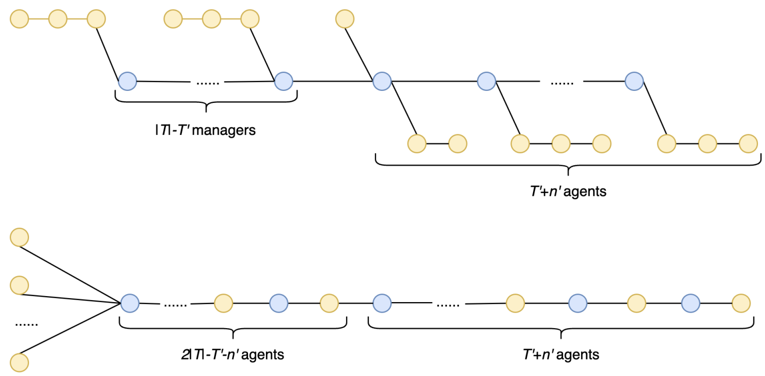


Figure 9: An optimal sequence $\pi^*(g_{cc})$ of g_{cc} (top) and the constructed sequence π' for g_{cp} (bottom).

agents in π' also form a line, it is easy to see that the total rewards must be lower in π' than in $\pi^*(g_{cc})$. Note that both π' and $\pi^*(g_{cc})$ yield a minimally connected monitoring network with $n-1$ links. Since π' is not necessarily an efficient sequence of g_{cp}, the total cost must be lower in g_{cp} than in g_{cc}, i.e., $C(g_{cp}) < C(g_{cc})$, when $\hat{f}$ is fixed and $|T|$ is sufficiently large. $\square$

Since $C(g_{cc}) < C(g_{hc})$ for all n, Lemma 8 implies that $C(g_{cp}) < C(g_{cc}) < C(g_{hc})$ when $\hat{f}$ is fixed and $|T|$ is sufficiently large.

Second, fix $|T|$ and let $\hat{f} \to \infty$. Consider an efficient sequence of g_{cp}, denoted $\tilde{\pi}(g_{cp})$. For any subordinate i, he must move either before the first manager, or after the last manager, or between two managers in $\tilde{\pi}(g_{cp})$. Note that if i moves between two managers, then i must be directly linked to each of the two managers. Suppose not, then i is directly linked to only one manager, since $\tilde{g}$ is connected for g_{cp}. Then, it is profitable to move i to either before the first manager or after the last manager, thereby enhancing transparency without increasing link costs, a contradiction. By a similar argument as Lemma 7, the number of subordinates after the last manager in $\tilde{\pi}(g_{cp})$ must be of lower order of n, as $\hat{f} \to \infty$, since these agents' reward equals $[p(n) - p(n-1)]^{-1}$. Similarly, as $\hat{f} \to \infty$, the average reward under $\pi^*(g_{hc})$, $V^*_{hc}/n \to [p(n) - p(0)]^{-1}$. Note that $\pi^*(g_{hc})$ yields a minimally connected monitoring network with n links; thus, $C(g_{hc})/n \to [p(n) - p(0)]^{-1} + c$. Suppose as $\hat{f} \to \infty$, the total number of subordinates between two managers in $\tilde{\pi}(g_{cp})$ is larger than αn for some constant $\alpha \in (0, 1)$. Since each reward is at least $[p(n) - p(0)]^{-1}$, $\lim_{n \to \infty} C(g_{cp})/n \geq [p(n) - p(0)]^{-1} + (1 + \alpha)c$. Thus, $C(g_{hc}) < C(g_{cp})$. By contrast, suppose as $\hat{f} \to \infty$, the total number of subordinates between two managers in $\tilde{\pi}(g_{cp})$ is of lower order of n. Then, given $\hat{f}$, construct a sequence π' in g_{hc}, such that $|T| - 1$ type-I teams move before the top manager, and 1 type-III team moves after him, as illustrated in Figure 10. Note that the agents after the top manager form a line and account for $1/|T|$ of the population, as $\hat{f} \to \infty$. On the other hand, in $\tilde{\pi}(g_{cp})$, the

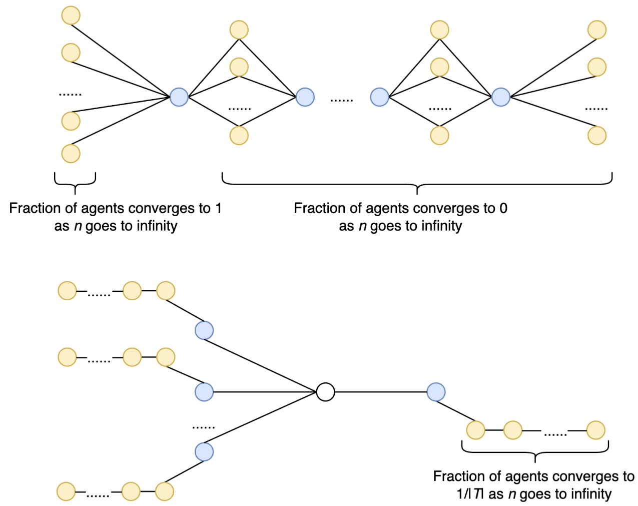


Figure 10: An efficient sequence $\tilde{\pi}(g_{cp})$ of g_{cp} (top) and the constructed sequence π' for g_{hc} (bottom).

agents after the first manager account for an infinitesimal fraction of the population, with most agents moving before the first manager, as $\hat{f} \to \infty$. Note that the total rewards under $\tilde{\pi}(g_{cp})$ are higher than they would be if the agents after the first manager form a line, which would be higher than the total rewards under π', since in π' the line after the top manager is significantly longer. Since an efficient sequence of g_{hc} is connected, $C(g_{hc})$ must be lower than the resulting total cost by implementing π' and cutting the link of the last agent in π', which yields $n-1$ links as $\tilde{\pi}(g_{cp})$. Thus, $C(g_{hc}) < C(g_{cp})$. In summary, $C(g_{hc}) < C(g_{cp})$, as $\hat{f} \to \infty$. Since $C(g_{cc}) < C(g_{hc})$ for all n, we have that $C(g_{cc}) < C(g_{hc}) < C(g_{cp})$ when $|T|$ is fixed and $\hat{f}$ is sufficiently large. Thus, the proposition is proven. □

Proof of Proposition 12

Proof. Statement (i) and the first part of statement (ii) are straightforward. We now show that $S(g_{cp}) > S(g_{cs})$. By direct calculation, we have

$$\begin{aligned}
\frac{|g_{cp}|}{2} &= |T|^2\hat{f} + \frac{|T|(|T|-1)}{2};\\
\frac{|g_{cs}|}{2} &= |T|\hat{f} + \frac{|T|(|T|-1)}{2};\\
\frac{|\tilde{g}(\pi^*(g_{cp}); g_{cp})|}{2} &\in [|T|(\hat{f}+1)-1, 2|T|\hat{f});\\
\frac{|\tilde{g}(\pi^*(g_{cs}); g_{cs})|}{2} &= |T|(\hat{f}+1)-1.
\end{aligned}$$

In particular, $|\tilde{g}(\pi^*(g_{cp}); g_{cp})|/2 < 2|T|\hat{f}$ because the transitive reduction $\pi^*(g_{cp})$ contains no link between managers and strictly less than 2 links per subordinate.

Now, define $\tilde{S}(g) \coloneqq |g|/|\tilde{g}(\pi^*; g)|$. Note that for two networks g and $\hat{g}$, $S(g) > S(\hat{g})$ if and only if $\tilde{S}(g) > \tilde{S}(\hat{g})$. Then, we have

$$\frac{\tilde{S}(g_{cp})}{\tilde{S}(g_{cs})} > \frac{|T|^2\hat{f} + \frac{|T|(|T|-1)}{2}}{|T|\hat{f} + \frac{|T|(|T|-1)}{2}} \frac{|T|(\hat{f}+1)-1}{2|T|\hat{f}} = \frac{(|T|\hat{f} + \frac{|T|-1}{2})(|T|\hat{f}+|T|-1)}{2|T|\hat{f}(\hat{f}+\frac{|T|-1}{2})}.$$

Given $|T| \geq 2$ and $\hat{f} \geq 2$, we have

$$\begin{aligned}
&(|T|\hat{f} + \frac{|T|-1}{2})(|T|\hat{f}+|T|-1) - 2|T|\hat{f}(\hat{f}+\frac{|T|-1}{2}) \\
&= |T|(|T|-2)\hat{f}^2 + \frac{(|T|\hat{f}+|T|-1)(|T|-1)}{2} > 0,
\end{aligned}$$

This implies that $S(g_{cp}) > S(g_{cs})$.

Then, we turn to statement (iii). Similarly, we have

$$\begin{aligned}
\frac{|g_{cc}|}{2} &= \frac{\hat{f}(\hat{f}-1)}{2}|T| + |T|\hat{f} + \frac{|T|(|T|-1)}{2} \\
\frac{|g_{hc}|}{2} &= \frac{\hat{f}(\hat{f}-1)}{2}|T| + |T| \\
\frac{|\tilde{g}(\pi^*(g_{cc}); g_{cc})|}{2} &= |T|(\hat{f}+1) - 1 \\
\frac{|\tilde{g}(\pi^*(g_{hc}); g_{hc})|}{2} &= |T|(\hat{f}+1).
\end{aligned}$$

It follows that

$$\begin{aligned}
\lim_{|T|\to\infty} \frac{\tilde{S}(g_{cp})}{\tilde{S}(g_{cc})} &> \lim_{|T|\to\infty} \frac{|T|^2\hat{f} + \frac{|T|(|T|-1)}{2}}{\frac{\hat{f}(\hat{f}-1)}{2}|T| + |T|\hat{f} + \frac{|T|(|T|-1)}{2}} \frac{|T|(\hat{f}+1)-1}{2|T|\hat{f}} = \frac{2\hat{f}+1}{2} > 1 \\
\lim_{\hat{f}\to\infty} \frac{\tilde{S}(g_{cp})}{\tilde{S}(g_{hc})} &\leq \lim_{\hat{f}\to\infty} \frac{|T|^2\hat{f} + \frac{|T|(|T|-1)}{2}}{\frac{\hat{f}(\hat{f}-1)}{2}|T| + |T|} \frac{|T|(\hat{f}+1)}{|T|(\hat{f}+1)-1} = 0 < 1 \\
\lim_{|T|\to\infty} \frac{\tilde{S}(g_{cs})}{\tilde{S}(g_{hc})} &= \lim_{|T|\to\infty} \frac{|T|\hat{f} + \frac{|T|(|T|-1)}{2}}{\frac{\hat{f}(\hat{f}-1)}{2}|T| + |T|} \frac{|T|(\hat{f}+1)}{|T|(\hat{f}+1)-1} = \infty > 1 \\
\lim_{\hat{f}\to\infty} \frac{\tilde{S}(g_{cs})}{\tilde{S}(g_{hc})} &= \lim_{\hat{f}\to\infty} \frac{|T|\hat{f} + \frac{|T|(|T|-1)}{2}}{\frac{\hat{f}(\hat{f}-1)}{2}|T| + |T|} \frac{|T|(\hat{f}+1)}{|T|(\hat{f}+1)-1} = 0 < 1.
\end{aligned}$$

This establishes statement (iii). Thus, the proposition is proven. □

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