Selling Signals

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Abstract

This paper studies a signaling model in which a strategic player can manipulate the signaling cost. A seller chooses a price schedule for a product, and a buyer with a hidden type chooses how much to purchase as a signal to receivers. When receivers observe the price schedule, the seller charges monopoly prices, and the buyer purchases less than the first-best. In contrast, when receivers do not observe the price schedule, the demand for signals is more elastic. Thus, the seller charges lower prices, and the buyer purchases more than in the observed case; those of the highest types purchase more than the first-best. The model suggests that price transparency benefits the seller but harms the buyer. The model can be applied to schools choosing tuition, retailers selling luxury goods and media companies selling advertising messages.

Keywords: Signaling, Screening, Signal Jamming, Price Transparency

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1 Introduction

Signaling is prevalent in various markets. Whereas in classic signaling models the sender's preference depends only on his intrinsic type, in many vertical markets in which signaling prevails, the signaling cost—thus the sender's preference—also depends on the choice made by an upstream strategic player. For example, when a student obtains education to signal his ability, the university sets the tuition; when a consumer purchases a luxury good to signal his wealth, the retailer chooses the price; when a firm incurs advertising expenses to signal its product's quality, the media company determines the cost of advertising messages.

A key observation is that since the signaling cost is endogenous, how receivers interpret and respond to the sender's signal depends on whether they observe the upstream player's choice. Consider a seller choosing the price of a product that creates signaling value for the customers, as in the above instances. How does receivers' information about the price affect the seller's pricing strategy? How does such information affect the degree of signaling?

In this paper, we characterize the optimal price schedule for a seller facing a buyer who is endowed with a hidden type and chooses how much to purchase as a signal to receivers. The equilibrium depends critically on whether receivers observe the price schedule. When receivers observe the price schedule, the seller internalizes the buyer's signaling incentive when screening the buyer, leading to a downward distortion in quantity. In contrast, when receivers do not observe the price schedule, the buyer is more sensitive to price changes, since receivers will attribute a difference in quantity to buyer preference heterogeneity. This implies that the demand for the product is more elastic, and thus, the seller lowers prices. In equilibrium, the buyer chooses a higher quantity and obtains higher utility than in the observed case, whereas the seller gains lower profits than in the observed case.

This paper has meaningful implications for the price transparency of signaling goods. In the case of job market signaling, our model suggests that education is more costly and students are worse off when employers observe the net prices for school than otherwise. This implies that policies that improve the transparency of the net prices at colleges and universities, e.g., U.S. Code § 1015a,[1] may *unintentionally* raise education expenses and harm students. This is because these policies allow schools to commit to high prices and not dilute the signaling value of a high-cost education by means of fee waivers or financial aid.

In addition, our model suggests that a signaling good yields higher profits if the price is

[1] Since 2011, American colleges and universities have been required to provide reasonable estimates of the net prices, including tuition, miscellaneous fees and personal expenses, that students will pay for school. See "U.S. Code § 1015a - Transparency in college tuition for consumers" for details.

more transparent. This is consistent with real-world business practices. For example, luxury brands, such as Louis Vuitton, Tiffany and Hermes, enjoy a reputation of never or very rarely being on sale. It is also reported that Burberry, Britain's largest luxury label, burned £28.6 million of clothing and cosmetics in fiscal year 2017 to prevent unwanted items being sold at a steep discount.[2] These strategies help the sellers better commit to high prices, thereby maintaining the signaling values of luxury goods. In the advertising industry, the high costs of each year's Super Bowl commercials are widely reported, thereby enhancing the signaling value of these costly commercials; in China, the TV station CCTV broadcasts the auctions for its popular TV show commercials to accentuate their signaling values.

For the purpose of exposition, we present our model à la Spence (1973) with a school selling productive education to a worker. As a starting point, we revisit Spence's model by fixing tuition at zero, as if schools were competitive and set the price equal to the marginal cost. In the least-cost separating equilibrium, all types except the lowest one choose more education than the first-best, as they attempt to separate themselves from lower types.

In Section 3, we introduce the school and study the case wherein employers observe the tuition scheme. In the school-optimal separating equilibrium (which is also the least-cost separating equilibrium), all types except the highest one choose less education than the first-best. This result contrasts with that of Spence's model. The downward distortion is due to screening. With a cost advantage in education, a higher type can secure higher utility than a lower type by imitating the latter, meaning that the worker can extract information rents from the school. This induces the school to under-supply education.

While this mechanism is similar to screening models such as Mussa and Rosen (1978), our model also incorporates signaling, which can mitigate the downward distortion caused by screening. To illustrate, suppose that employers can observe the worker's ability, thereby eliminating signaling. When a higher type imitates a lower type, he not only incurs a lower total cost than the latter but also obtains a higher wage due to his higher ability. The second effect means that the worker can extract more information rents from the school; thus, the screening distortion is worse compared to when signaling is present.

In Section 4, we turn to the case wherein employers do not observe the tuition scheme. In the school-optimal separating equilibrium (which is also the least-cost separating equilibrium), the school sets lower tuition rates and the worker chooses more education than when employers observe the tuition scheme. This difference is driven by a *signal jamming effect*. Because employers cannot observe the actual cost of education, they do not know whether

[2]See "Burberry to stop burning clothing and other goods it can't sell," *New York Times* (Sept. 6, 2018).

a difference in education level is due to a tuition change or worker cost heterogeneity. For example, suppose that the school lowers tuition so that the worker obtains more education than in the initial state. When employers observe the tuition scheme, they cut wages, since any education level now corresponds to a lower-ability worker. This dampens the worker's demand for additional education. In contrast, when employers do not observe the tuition scheme, they do not adjust wages despite that tuition changes. Therefore, the demand for education is more elastic, making the price cut more profitable. In equilibrium, employers correctly anticipate the school's choice and offer lower wages, as education is inflated. This reduces the worker's willingness to pay, and thus, the school achieves lower profits.

Since the school is worse off when employers do not observe the tuition scheme, one may wonder why the school does not disclose tuition to employers. The reason is that the school cannot credibly announce the price absent intervention such as mandatory disclosure, since the school has an inventive to secretly cut prices. Such an observation may explain the fact that while the listed tuition at American colleges and universities is rising, these schools offer students various and inclusive forms of financial aid.[3] The rationale is that employers cannot easily observe the details of such financial aid and thus do not know the actual cost of education. By raising the published tuition while simultaneously reducing the undisclosed net prices through stipends, schools persuade employers that their students are smarter than is actually the case, thereby allowing the schools to collect higher revenues from students.

In section 5, we consider welfare. We show that when signaling is sufficiently intense, social welfare is higher when the tuition scheme is observed by employers than otherwise. This is because in the observed case signaling mitigates the screening distortion to a large extent, whereas in the unobserved case cheaper tuition leads to a large fraction of higher types who over-invest in education. Moreover, when signaling is intense, both cases yield higher social welfare than Spence's model in which schools are competitive. This implies that introducing competition among signal sellers is not necessarily socially beneficial.

Finally, in Section 6, we elaborate how to apply our model to the cases of conspicuous consumption and advertising, and explore some extensions. Section 7 concludes our paper. All omitted proofs are provided in the Appendix.

[3]According to the reports by the College Board (www.collegeboard.org): "from 2007-08 through 2010-11, the percentage of institutional grant aid that helped to meet students' financial need at private nonprofit four-year colleges and universities ranged from a low of 90% to a high of 93%" (*Trends in Student Aid 2011*, the College Board); "between 2008-09 and 2013-14, the $3,800 increase (in 2013 dollars) in average institutional grant aid for first-time full-time students at private bachelor's institutions covered 95% of the $4,000 increase in tuition and fees" (*Trends in Student Aid 2016*, the College Board).

1.1 Related Literature

This paper is most closely related to the literature on signaling. The paper contributes to the literature on signaling games by allowing a strategic player to affect signaling cost. In classic signaling models (e.g., Spence, 1973; Leland and Pyle, 1977; Riley, 1979; Milgrom and Roberts, 1986; Bagwell and Riordan, 1991), with exogenous signaling cost, signaling activity gives rise to over-investment in costly actions. Spence (1974), Ireland (1994) and Andersson (1996) suggest taxing signaling activity to undo the signaling effect to restore the first-best. The associated tax scheme is thus the welfare-maximizing tax on signals. In our model, when receivers observe the price schedule, we solve for the profit-maximizing tax on signals, which "over-taxes" signaling and causes a downward distortion in quantity.

The paper is also closely related to the literature on screening. Screening models, such as Mussa and Rosen (1978) and Maskin and Riley (1984), typically assume that buyers derive intrinsic utility from consuming the seller's product. Our model differs in the sense that the product has further a signaling value, and a buyer's utility depends on the information that the product conveys. Rayo (2013) also considers the optimal monopoly pricing to sell signals, assuming that the seller's mechanism is observed by receivers. Whereas we assume additive separability in receivers' actions (e.g., wages) and the buyer's type (e.g., ability), Rayo's adopts a multiplicative structure, and employs novel screening techniques to characterize which types should be pooled into the same level of signal. The contribution of our paper is to study the case in which receivers cannot observe the seller's mechanism, and comparing this to the observed case and a variety of other benchmarks. This enables us to assess how price transparency affects the degree of signaling and welfare. Calzolari and Pavan (2006) study information disclosure in a sequential screening model. They show that the upstream principal leaves more rents to the agent if she discloses information about the agent's type to the downstream principal. In our model, the seller leaves more rents to the buyer if receivers can observe the buyer's type, which is perfect information disclosure. In contrast to their model, the disclosure of the buyer's type creates no value for our competitive receivers.

The unobserved tuition case belongs to the class of signal jamming models proposed by Fudenberg and Tirole (1986). For example, in Holmström (1999), the labor market cannot distinguish the impact of the worker's ability from that of his effort on output. In response, the worker works harder to improve the market's perception of his ability. In comparison, in our model, the labor market cannot distinguish the impact of the worker's ability from that of tuition on education level. Thus, the school has an incentive to secretly cut tuition, thereby improving the market's perception and stimulating demand. In Chan et al. (2007), a school has an incentive to inflate grades to improve the market's perception of its students. They show that grade inflation features strategic complements when the qualities of students are correlated across schools. In contrast to their model, our model incorporates screening in addition to signaling, as the school cannot observe its students' abilities. Zubrickas (2015) studies the optimal grading policy when a school cannot observe its students' abilities and the labor market has myopic beliefs over the school's grading policies.

Finally, our paper relates closely to the literature on intermediate price transparency. Inderst and Ottaviani (2012) shows how product providers compete through commissions paid to consumer advisers. Commissions bias advice; thus, an increase in a firm's commission reduces consumers' willingness to pay if they observe the commission. Analogously, in our model, cheaper tuition reduces the signaling value of education, and thus, tuition cuts are less effective at stimulating demand than they would be otherwise when employers observe tuition. In Janssen and Shelegia (2015), a manufacturer chooses a wholesale price, retailers choose retail prices, and consumers search for the best deal. They argue that retailers are less sensitive to wholesale price changes when consumers do not observe the price than otherwise, as uninformed consumers are more likely to keep searching when the retail price raises. By contrast, in our model the worker is more sensitive to tuition changes when employers do not observe the tuition scheme than otherwise, as uninformed employers will have better (worse) beliefs over the worker's ability if they observe a higher (lower) education level.

2 The Model

For ease of exposition, in this section, we present our model in conformity with the seminal work of Spence (1973) with productive education. In Section 6, we discuss how to apply the general model to the cases of conspicuous consumption and advertising.

Players and actions. There is a school, a worker and multiple identical and competing firms, also referred to as *the labor market*. At the beginning of the game, the school chooses a tuition scheme $T : \mathbb{R}_+ \to \mathbb{R}_+$, which specifies the tuition rate at each education level z. Then, the worker decides how much education to purchase from the school based on the tuition scheme. For simplicity, we do not explicitly model firms' actions; rather, we directly assume that they offer the worker a wage equal to his expected productivity (see below).

The worker's productivity depends on his ability (*type*) θ and his education choice z. Specifically, θ is a random variable, which distributes over the interval $[\underline{\theta}, \bar{\theta}]$, according to a distribution function $F(\theta)$ with a positive density function $f(\theta)$. Denote by $Q(z, \theta)$ the productivity of a type-θ worker having education level z. We assume that $Q(z,\theta)$ is twice differentiable and increasing in both arguments. Formally, $Q_z(z,\theta)$, $Q_\theta(z,\theta) > 0$ if $z > 0$. We also assume that a worker with no education has zero productivity irrespective of his ability; that is, $Q(0,\theta) \equiv 0$. We consider this assumption realistic since many jobs require a minimal education level. For example, a lawyer candidate must graduate from a law school, and medical school education is prerequisite for being a licensed practitioner of medicine.

Information. The worker's education level is publicly observed. However, neither the school nor the labor market observes the worker's ability, but both know its distribution. In this paper, we mainly study two variants of the model: in the *observed* case, the tuition scheme is observed by the labor market; in the *unobserved* case, it is unobserved by the labor market. In each case, based on the available information, the labor market chooses a wage schedule $W : \mathbb{R}_+ \to \mathbb{R}_+$, which specifies the wage for each education level z.

Payoffs. We normalize the school's cost of providing education to zero. Suppose that the school chooses some tuition scheme T; then, let $z(\theta; T)$ be the education level chosen by a type-θ worker under T. Given the tuition scheme T and the wage schedule W, a type-θ worker who chooses education level z has utility

$$U(z,\theta) \coloneqq W(z) - C(z,\theta) - T(z),$$

where $C(z,\theta)$ is the worker's cost of effort for education. We assume that $C(z,\theta)$ is twice differentiable, increasing and strictly convex in z, and unbounded: $C_z(z,\theta) > 0$ if $z > 0$, and $C_{zz}(z,\theta) > k$ for some $k > 0$. Moreover, the standard *single-crossing property* holds: $C_{z\theta}(z,\theta) < 0$ if $z > 0$. This condition captures the feature that a higher-ability worker has lower marginal effort costs than a lower-ability worker. We also normalize $C(0,\theta)$ to 0 for all $\theta \in [\underline{\theta}, \bar{\theta}]$. This implies that, combined with $C_z(z,\theta) > 0$ and $C_{z\theta}(z,\theta) < 0$ if $z > 0$, $C_\theta(z,\theta) < 0$ if and only if $z > 0$. Finally, we assume that the worker can obtain a zero-utility outside option by acquiring no education and not entering the labor market.

First-best benchmark. Define $S(z,\theta)$ as the social surplus function, i.e.,

$$S(z,\theta) \coloneqq Q(z,\theta) - C(z,\theta).$$

Assume that $S(z,\theta)$ is strictly quasiconcave in z and has a unique maximizer $z^{fb}(\theta) \geq 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Then, the first-order condition implies that

$$S_z(z^{fb}(\theta),\theta) = Q_z(z^{fb}(\theta),\theta) - C_z(z^{fb}(\theta),\theta) = 0. \tag{2.1}$$

To ensure that $z^{fb}(\theta)$ is increasing, we assume further that $S_{z\theta}(z,\theta) > 0$ if $z > 0$. Then, the monotonicity holds according to Milgrom and Shannon (1994, Theorem 4). It is also readily confirmed that $S(z^{fb}(\theta),\theta)$ is increasing in θ.

Equilibrium. We use *perfect Bayesian equilibrium* as the solution concept throughout the paper. In the observed case, an equilibrium consists of a tuition scheme T^o and conditional on any scheme T, an education function $z^o(\theta;T)$ and a wage schedule $W^o(z;T)$, such that

(*i*) For each T, the following holds: (a) given $W^o(z;T)$, $z^o(\theta;T)$ maximizes $U(z,\theta)$; (b) $W^o(z;T) = \mathbb{E}[Q(z,\theta)]$ such that the labor market's posterior belief about the worker's ability, or simply *the market belief*, is updated using Bayes' rule whenever possible.

(*ii*) Given $z^o(\theta;T)$, T^o maximizes the school's expected profit, i.e.,

$$T^o \in \underset{T}{\operatorname{argmax}} \int_{\underline{\theta}}^{\bar{\theta}} T(z^o(\theta;T))dF(\theta).$$

In the unobserved case, the market's inference is independent of the actual tuition scheme but is conditional on a *conjectured* scheme; in equilibrium, the conjecture is correct. In this case, an equilibrium consists of a tuition scheme T^u and a wage schedule W^u, and conditional on any T, an education function $z^u(\theta;T)$, such that

(*i*) Given W^u, for each T, $z^u(\theta;T)$ maximizes $U(z,\theta)$; $W^u(z) = \mathbb{E}[Q(z,\theta)]$ such that the market belief is updated using Bayes' rule whenever possible.

(*ii*) Given $z^u(\theta;T)$, T^u maximizes the school's expected profit, i.e.,

$$T^u \in \underset{T}{\operatorname{argmax}} \int_{\underline{\theta}}^{\bar{\theta}} T(z^u(\theta;T))dF(\theta).$$

Note that the equilibrium conditions have a prominent difference between the observed and unobserved case: in the unobserved case, the market belief needs to be correct only on the equilibrium path, whereas in the observed case, the market belief has to be correct following every tuition scheme that is chosen by the school.

Equilibrium selection. For both the observed and unobserved case, while there possibly exist multiple equilibria, we focus on the *school-optimal separating equilibrium*, that is, the equilibrium that yields the highest payoff for the school, provided that on the equilibrium path, $z(\theta)$ is one-to-one if $z > 0$. To ensure that a separating equilibrium exists, we impose a standard regularity condition following the literature.

Assumption 1. *$C_{z\theta\theta}(z,\theta) \geq 0$ and $F(\theta)$ has a non-decreasing hazard rate.*

The reason that we select the school-optimal separating equilibrium is because in Spence's model, the unique equilibrium that survives the D1 refinement (Banks and Sobel, 1987) is the *least-cost separating equilibrium* (Riley, 1979) in which $z(\theta)$ is one-to-one and the lowest type $\underline{\theta}$ chooses the first-best $z^{fb}(\underline{\theta})$. In the school-optimal separating equilibrium, given the equilibrium tuition scheme, the continuation game constitutes the least-cost separating equilibrium, thereby allowing us to compare the associated equilibrium predictions with that of Spence's model. Moreover, in a discrete-type version of the model, a pooling equilibrium, in which all participating types choose an identical education level, does not exist in either case whenever the fraction of the highest type is sufficiently large.[4] Therefore, we consider the school-optimal separating equilibrium a reasonable equilibrium to study. In Sections 3 and 4, we shall discuss equilibrium selection in greater detail.

2.1 Direct Mechanisms

Appealing to the revelation principle, we consider direct mechanisms between the school and worker in both the observed and unobserved case. It is without loss of generality to adjust the timing as follows. First, the school offers a contract $\{z(\theta), T(\theta)\}$ to the worker. Then, the labor market publishes a wage schedule $W(z)$ based on the information available: in the observed case, it observes the contract; in the unobserved case, it does not. Finally, the worker reports a type to *only* the school.[5] Reporting a type $\hat{\theta}$, the worker obtains education level $z(\hat{\theta})$, pays tuition $T(\hat{\theta})$ and then receives wage $W(z(\hat{\theta}))$.

Worker's problem. In both cases, given a contract $\{z(\theta), T(\theta)\}$ and the associated wage schedule $W(z)$, a type-θ worker chooses a report $\hat{\theta}$ to maximize his utility

$$U(\hat{\theta}, \theta) := W(z(\hat{\theta})) - C(z(\hat{\theta}), \theta) - T(\hat{\theta}).$$

A contract with the associated wage schedule is *incentive compatible* if the worker is willing to truthfully report his type and is *individually rational* if the worker obtains a non-negative utility level. A type-θ worker's equilibrium payoff is represented by $U(\theta) := U(\theta, \theta)$.

[4]Suppose that, in either the observed or unobserved case, a pooling equilibrium exists such that all participating types choose the same education; then, the equilibrium wage is a constant which equals the average productivity, and the equilibrium tuition is a fixed fee which makes the lowest participating type indifferent. But whenever the fraction of the highest type is sufficiently large, it is optimal for the school to serve only the highest type and exclude all lower types, leading to a contradiction.

[5]If reports are public, then the set of outcomes that can be implemented by truthful direct mechanisms is smaller than what can be obtained via an indirect mechanism wherein the worker only chooses education.

School's problem. In the observed case, the school chooses a contract to maximize its expected profit subject to incentive compatibility (IC), individual rationality (IR), and the market belief being correct. In the unobserved case, given the wage schedule, the school chooses a contract to maximize its expected profit subject to IC and IR constraints.

Preliminaries. In both cases, an allocation $\{z(\theta), U(\theta)\}$ is *implementable* if it is incentive compatible and individually rational. Appealing to Mas-Colell, Whinston, and Green (1995, Proposition 23.D.2), we characterize all implementable allocations by the following lemma.

Lemma 1. *In both cases, an allocation* $\langle z(\theta), U(\theta)\rangle$ *is implementable if and only if*

(i) $z(\theta)$ *is non-decreasing.*

(ii) Define $\theta_0 := \inf\{\theta | z(\theta) > 0\}$*; then, for* $\theta > \theta_0$,

$$U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} -C_\theta(z(s), s)ds$$

subject to $U(\theta_0) \geq 0$.

By Lemma 1, we can rewrite the school's problem for both cases. Note that IC means that $T(\theta) = W(z(\theta)) - C(z(\theta), \theta) - U(\theta)$ and that $U(\theta_0)$ is optimally set to 0. Substituting and integrating by parts, the school's problem can be stated as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ W(z(\theta)) - C(z(\theta), \theta) + \frac{1 - F(\theta)}{f(\theta)} C_\theta(z(\theta), \theta) \right\} dF(\theta) \tag{2.2}$$

subject to $z(\theta)$ being non-decreasing.

In the observed case, correctness of the market belief means that $W(z) = \mathbb{E}[Q(z, \theta)|z(\theta)]$ for any implementable allocation $z(\theta)$ that the school chooses. Then, from the law of total expectation, Program (2.2) is equivalent to

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ S(z(\theta), \theta) + \frac{1 - F(\theta)}{f(\theta)} C_\theta(z(\theta), \theta) \right\} dF(\theta) \tag{2.3}$$

subject to $z(\theta)$ being non-decreasing. It suffices to solve Program (2.3) for the equilibrium characterization of the observed case. If the solution $z(\theta)$ is increasing over $[\theta_0, \bar{\theta}]$, then we obtain the school-optimal separating equilibrium.

In the unobserved case, without loss of generality, the school chooses an allocation $z(\theta)$, while simultaneously, the labor market chooses a wage schedule $W(z)$. Then, the equilibrium conditions can be simplified as follows: (i) given $W^u(z)$, $z^u(\theta)$ solves the school's problem in (2.2); (ii) $W^u(z) = \mathbb{E}[Q(z, \theta)]$ such that the market belief is updated using Bayes' rule. In the case of multiple equilibria, we select the school-optimal separating equilibrium.

2.2 Spencian Job Market Signaling

As a reference point, we briefly revisit Spence's signaling game in which tuition is fixed at zero. One could interpret such a benchmark as the case in which schools are competitive and thus choose tuition equal to the marginal cost. In this case, an equilibrium consists of an education function $z^s(\theta)$ and a wage schedule $W^s(z)$, such that (i) given $W^s(z)$, $z^s(\theta)$ maximizes $U(z,\theta)$; (ii) $W^s(z) = \mathbb{E}[Q(z,\theta)]$ with the market belief updated by Bayes' rule. We study the least-cost separating equilibrium. According to Mailath (1987), the equilibrium exists, such that the education function $z^s(\theta)$ is given by the initial value problem (IVP):

$$Q_z(z^s(\theta),\theta) + Q_\theta(z^s(\theta),\theta) \cdot \theta^{s\prime}(z^s(\theta)) - C_z(z^s(\theta),\theta) = 0, \tag{2.4}$$

with $z^s(\underline{\theta}) = z^{fb}(\underline{\theta})$, where $\theta^s(z)$ is the inverse function of $z^s(\theta)$. Moreover, it can be verified that $z^s(\theta)$ is increasing over $[\underline{\theta},\bar{\theta}]$, and thus, $W^s(z^s(\theta)) = Q(z^s(\theta),\theta)$ for all $\theta \in [\underline{\theta},\bar{\theta}]$.

Note that the first two terms on the left-hand side (LHS) of (2.4) are the total derivative of $W^s(z)$. In particular, the second term is non-negative given the monotonicity of $z^s(\theta)$. Since $S(z,\theta)$ is strictly quasiconcave, comparing (2.4) with (2.1) implies that $z^s(\theta) \geq z^{fb}(\theta)$ for all $\theta \geq \underline{\theta}$, with equality holding at $\underline{\theta}$ only. This result means that in Spence's signaling game, the worker chooses more education than the first-best; that is, the worker's signaling activity leads to over-education. The intuition is well-understood. Under complete information, the marginal benefit of education is its marginal contribution to human capital. In contrast, when ability is privately known, in addition to the human capital effect, there is a *signaling effect*; that is, a higher education level makes the labor market regard the worker as having higher ability. Thus, the marginal benefit of education is higher than under complete information.

3 Labor Market Observes Tuition

Starting with this section, we take the school's strategic behavior into account. Here, we consider the case in which the labor market observes the tuition scheme. From Section 2.1, it suffices to solve Program (2.3) for the equilibrium characterization. It is heuristic to regard the integrand in (2.3) as the school's marginal profit in the observed case. Define

$$MP^o(z,\theta) \coloneqq S(z,\theta) + \frac{1-F(\theta)}{f(\theta)} C_\theta(z,\theta).$$

As is standard in the literature, we solve the school's problem with the monotonicity constraint relaxed. This is equivalent to pointwise optimization for $MP^o(z,\theta)$. Inspired by Martimort and Stole (2009), we say that the school's marginal profit in the observed case is *regular* if $MP^o(z,\theta)$ is strictly quasiconcave in z and $MP^o_z(z,\theta)$ is increasing in θ. Given Assumption 1, regularity holds. Thus, $MP^o(z,\theta)$ has a unique maximizer $z^*(\theta)$, which is increasing. Note that $z^*(\theta)$ might be negative for some region of θ; as such, we set $z(\theta)$ to 0 instead of $z^*(\theta)$. Hence, $MP^o(z(\theta),\theta)$ is non-decreasing in θ and is non-negative. The cutoff type θ^o_0 is thus either the maximal root of $MP^o(z(\theta),\theta) = 0$ if it exists, or $\underline{\theta}$ otherwise. In summary, the optimal allocation $z^o(\theta)$ is given by

$$z^o(\theta) = \begin{cases} z^*(\theta) & \text{if } \theta \geq \theta^o_0 \\ 0 & \text{otherwise.} \end{cases} \tag{3.1}$$

To complete the characterization, back out $z^o(\theta)$'s inverse function $\theta^o(z)$ on $[\theta^o_0, \bar{\theta}]$ given its monotonicity. Plugging $\theta^o(z)$ into $Q(z,\theta)$ yields the wage schedule W^o on $[z^o(\theta^o_0), z^o(\bar{\theta})]$. Finally, the tuition scheme T^o on $[z^o(\theta^o_0), z^o(\bar{\theta})]$ is given by

$$T^o(z^o(\theta)) = S(z^o(\theta),\theta) - U(\theta) = S(z^o(\theta),\theta) + \int_{\theta^o_0}^{\theta} C_\theta(z^o(s), s)ds. \tag{3.2}$$

For the off-path education levels, we assume without loss that the school sets exorbitantly high prices such that no type is willing to deviate to there in any case. Then, given T^o, the school-optimal separating equilibrium is also the least-cost separating equilibrium in the sense that the cutoff type chooses his full-information optimal quantity under the total cost function $C(z,\theta) + T^o(z)$. Moreover, since $z^o(\theta)$ coincides with the unconstrained optimizer $z^*(\theta)$ on path, this equilibrium is further *the school-optimal equilibrium.* We now summarize the equilibrium outcome of the observed case in the proposition below.

Proposition 1. *The school-optimal separating equilibrium exists. On the equilibrium path, the education function $z^o(\theta)$ is given by (3.1), the tuition scheme $T^o(z)$ is given by (3.2), and the wage schedule $W^o(z)$ equals $Q(z,\theta^o(z))$.*

Note that $MP^o_z(z,\theta)$ is less than $S_z(z,\theta)$, holding weakly on the boundary. Consequently, regularity implies that $z^o(\theta) \leq z^{fb}(\theta)$ on $[\theta^o_0, \bar{\theta}]$, with equality holding at $\bar{\theta}$ only. Particularly, if $\theta^o_0 > \underline{\theta}$, then $z^o(\theta) = 0$ for all $\theta \in [\underline{\theta}, \theta^o_0)$. To summarize, we have the following corollary:

Corollary 1. *In the observed case, the worker chooses less education than the first-best. Specifically, $z^o(\theta) \leq z^{fb}(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality on $(\underline{\theta}, \bar{\theta})$.*

Corollary 1 states that when the labor market observes the tuition scheme, education is under-supplied. This result stands in stark contrast to that of Spence's model. The altered equilibrium prediction results from the school's screening. Specifically, with a cost advantage in education, a higher-ability worker can secure higher utility than a lower-ability worker by imitating the latter. To incentivize truth-telling, the school has to leave information rents to the worker. This means that the marginal profit of education is less than the social surplus generated; thus, the school under-supplies education. In particular, an interval of types at the low end of the domain will be excluded from education if it is too costly to serve them.

3.1 Screening vs Signaling

While the equilibrium prediction for the observed case is due to the mechanism of monopoly screening, our model also contains signaling. Note that given the tuition scheme T^o, the subgame is essentially Spence's signaling game as if the worker had a cost function in the form of $C(z, \theta) + T^o(z)$. According to the same argument as in Section 2.2, the education levels in the observed case are distorted, due to signaling, above the full-information level with respect to the total cost of education. This fact reveals that the equilibrium outcome of the observed case results from the interaction between screening and signaling.

Corollary 1 implies that when both screening and signaling are present and exert the opposite effects—screening induces under-education, but signaling induces over-education—screening outweighs signaling. This is because as a Stackelberg leader, the school internalizes the worker's signaling incentive when screening his type. To see this, note that

$$T^{o\prime}(z) = W^{o\prime}(z) - C_z(z, \theta^o(z)) = \frac{d}{dz}\left[Q(z, \theta^o(z))\right] - C_z(z, \theta^o(z)).$$

Substituting this equation into the first-order condition of $MP^o(z, \theta)$, we have

$$T^{o\prime}(z) = Q_\theta(z, \theta^o(z)) \cdot \theta^{o\prime}(z) + \frac{1 - F(\theta^o(z))}{f(\theta^o(z))}\left[-C_{z\theta}(z, \theta^o(z))\right]. \tag{3.3}$$

On the right-hand side (RHS) of (3.3), the first term captures the signaling effect, and the second term is the marginal information rent extracted by the worker. Note that signaling induces over-education, which reduces the school's profit in two ways: on one hand, it lowers total surplus; on the other hand, it provides the worker with more information rents. Thus, the optimal tuition scheme must undo these two effects, as indicated by (3.3). In contrast, if the school were a welfare-maximizing social planner, it would only undo the signaling effect by levying Pigovian taxes (Spence, 1974). Denote by T^{fb} the welfare-maximizing tax on education. The marginal tax is equal to the signaling effect at the first-best, i.e.,

$$T^{fb\prime}(z) = Q_\theta(z, \theta^{fb}(z)) \cdot \theta^{fb\prime}(z), \tag{3.4}$$

where $\theta^{fb}(z)$ is the inverse function of $z^{fb}(\theta)$.[6] Since the second term on the RHS of (3.3) is positive, comparing (3.3) with (3.4) indicates that the profit-maximizing tax on education "over-taxes" signaling activity and thus leads to under-education.

To see how signaling makes a difference, consider the situation in which the labor market also observes the worker's ability without changing any other element of the model. In this case, the wage equals the actual productivity, and signaling is eliminated. This means that the worker's intrinsic value for education is the social surplus $S(z,\theta)$. Since $S_\theta = Q_\theta - C_\theta > 0$, a higher type can be seen as a higher-value buyer of education. Thus, the school has the same monopoly screening problem as in Mussa and Rosen (1978). Specifically, the school chooses a contract $\{z(\theta), T(z)\}$ to maximize its expected profit subject to IC and IR constraints. Analogous to Lemma 1, an allocation $\{z(\theta), U(\theta)\}$ is implementable if and only if (i) $z(\theta)$ is non-decreasing; (ii) $U(\theta_0) \geq 0$ and for $\theta > \theta_0$,

$$U(\theta) = U(\theta_0) + \int_{\theta_0}^{\theta} S_\theta(z(s), s) ds.$$

Thus, the school's problem in such a Mussa and Rosen's screening game can be stated as

$$\max_{z(\theta)} \int_{\theta_0}^{\bar{\theta}} \left\{ S(z(\theta),\theta) - \frac{1-F(\theta)}{f(\theta)} S_\theta(z(\theta),\theta) \right\} dF(\theta)$$

subject to $z(\theta)$ being non-decreasing.

Define analogously the school's marginal profit as

$$MP^{mr}(z,\theta) := S(z,\theta) - \frac{1-F(\theta)}{f(\theta)} S_\theta(z,\theta).$$

Similarly, we say that $MP^{mr}(z,\theta)$ is regular if it is strictly quasiconcave in z and $MP_z^{mr}(z,\theta)$ is increasing in θ.[7] Denote by $z^{mr}(\theta)$ and θ_0^{mr} the optimal allocation and the cutoff type in Mussa and Rosen's model, respectively. Suppose that $MP^{mr}(z,\theta)$ is regular, then one can characterize $z^{mr}(\theta)$ and θ_0^{mr} analogously to the observed case.

We shall examine how the allocation in Mussa and Rosen's model differs from that in the observed case. On the extensive margin, because $S_\theta > -C_\theta$, $MP^{mr}(z,\theta) \leq MP^{o}(z,\theta)$, with strict inequality for $\theta < \bar{\theta}$. Hence, if $\theta_0^{o} > \underline{\theta}$, then $\theta_0^{mr} > \theta_0^{o}$; that is, more types are excluded in Mussa and Rosen's model. On the intensive margin, if $Q_{z\theta} > 0$ on $[0, z^{fb}(\bar{\theta})]$,[8] then $z^{mr}(\theta) \leq z^{o}(\theta)$, with strict inequality on $[\theta_0^{o}, \bar{\theta})$, meaning that under-education is more serious in Mussa and Rosen's model. These findings are illustrated in Figure 1.

[6]Since the lowest type $\underline{\theta}$ chooses the first-best $z^{fb}(\underline{\theta})$ in equilibrium, he should be exempt from such tax; that is, $T^{fb}(z^{fb}(\underline{\theta})) = 0$. Then, directly integrating (3.4) yields the welfare-maximizing tax scheme T^{fb}.

[7]Given Assumption 1, $MP^{mr}(z,\theta)$ is regular if $S_{z\theta\theta} \leq 0$.

[8]This condition is not restrictive; indeed, given that $Q_\theta(z,\theta) > 0$ and $Q(0,\theta) \equiv 0$, we have $Q_{z\theta}(z,\theta) > 0$ on $[0, \bar{z}]$ for some $\bar{z} > 0$. Given this condition, $MP_z^{mr}(z,\theta) < MP_z^{o}(z,\theta)$ on $[0, z^{fb}(\bar{\theta})]$ for $\theta < \bar{\theta}$.

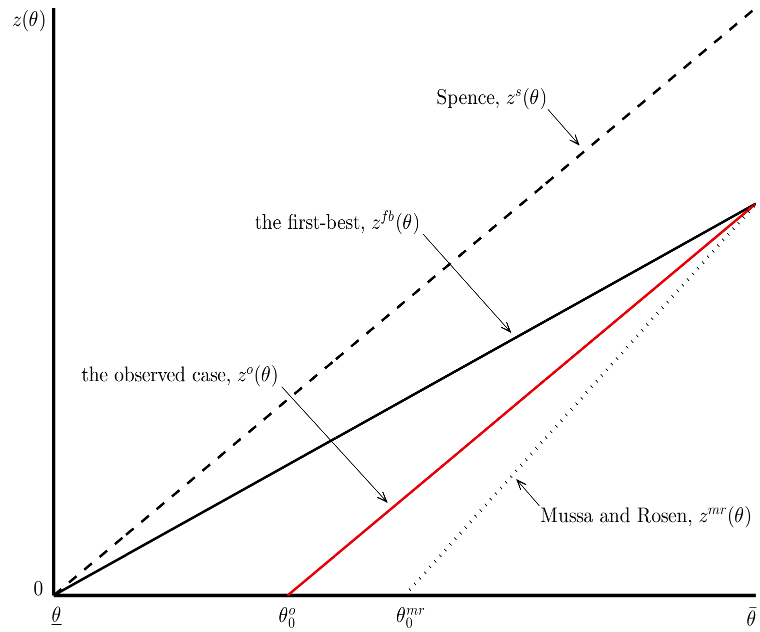

Figure 1: **Screening vs Signaling.** This figure compares $z^{mr}(\theta)$ with $z^o(\theta)$ over $[\underline{\theta}, \bar{\theta}]$ along with $z^{fb}(\theta)$ and $z^s(\theta)$. This figure assumes that $Q(z,\theta) = \theta z + z$, $C(z,\theta) = z^2 + z - \theta z$, and $\theta \sim U[0,1]$, such that $z^{fb}(\theta) = \theta$, $z^s(\theta) = 3\theta/2$, $z^o(\theta) = (3\theta - 1)/2$, and $z^{mr}(\theta) = 2\theta - 1$.

For welfare comparison, note that education is already under-supplied in the observed case, yet the downward distortion is larger in Mussa and Rosen's model; thus, the observed case has higher social welfare. Moreover, since $MP^{mr}(z^{mr}(\theta), \theta) \leq MP^o(z^o(\theta), \theta)$ with strict inequality on $[\theta_0^o, \bar{\theta})$ and $\theta_0^{mr} \geq \theta_0^o$, it is readily confirmed that the school's expected profit is also higher in the observed case. In summary, we have the following proposition:

Proposition 2. *If both $MP^o(z,\theta)$ and $MP^{mr}(z,\theta)$ are regular, and $Q_{z\theta} > 0$ on $[0, z^{fb}(\bar{\theta})]$, then under-education is greater when signaling is eliminated. Specifically, $z^{mr}(\theta) \leq z^o(\theta)$, with strict inequality on $[\theta_0^o, \bar{\theta})$; if $\theta_0^o > \underline{\theta}$, then $\theta_0^{mr} > \theta_0^o > \underline{\theta}$. Consequently, social welfare and the school's expected profit are strictly higher when signaling is present than otherwise.*

Proposition 2 indicates that signaling can mitigate the downward distortion caused by screening. Intuitively, when the labor market observes the worker's ability, if a higher type imitates a lower type by choosing the same education, he not only has a lower total cost than the latter but also obtains a higher wage due to his higher productivity. In contrast, when the labor market does not observe the worker's ability, the higher type can no longer directly reap the benefit from higher productivity, and thus, he acquires more education to signal his ability. The signaling incentive reduces the worker's willingness to imitate lower types. Therefore, the school leaves lower information rents to the worker when signaling is present, as we have the following inequality for all $\theta \in [\underline{\theta}, \bar{\theta}]$,

$$\underbrace{\frac{1-F(\theta)}{f(\theta)}\left[-C_\theta(z,\theta)\right]}_{\text{information rents with signaling}} \leq \underbrace{\frac{1-F(\theta)}{f(\theta)} S_\theta(z,\theta)}_{\text{information rents without signaling}}$$

which holds with equality at $\bar{\theta}$ only. Consequently, the screening distortion is mitigated.

Recall that in Spence's signaling game, signaling reduces social welfare, as it leads to over-education. In the observed case, by contrast, signaling raises social welfare because it mitigates the screening distortion. Thus, any instrument that attenuates signaling is socially beneficial in the Spencian world but harmful in the observed case. For example, students' grades substitute for their education levels in signaling. If grades become less informative, e.g., due to grade inflation, an increasingly common phenomenon at American colleges and universities,[9] then signaling through education will be enhanced, as students will attempt to separate themselves from others (Daley and Green, 2014). This reveals that coarse grading can be socially beneficial in the observed case by alleviating under-education,[10] while it is harmful in the Spencian world because it aggravates over-education.

4 Labor Market Does Not Observe Tuition

In this section, we turn to the case in which the labor market does not observe the tuition scheme. Given some wage schedule W, the school solves the problem in (2.2). Similarly to the observed case, we define the school's marginal profit in the unobserved case as

$$MP^u(z,\theta) := W(z) - C(z,\theta) + \frac{1-F(\theta)}{f(\theta)} C_\theta(z,\theta).$$

It is heuristic to call the last two terms the school's *virtual cost*, and we define

$$G(z,\theta) := C(z,\theta) - \frac{1-F(\theta)}{f(\theta)} C_\theta(z,\theta).$$

In doing so, we establish an auxiliary game analogous to Spence's signaling game, in which the worker's cost function is given by $G(z,\theta)$ and utility function by $MP^u(z,\theta)$.

This analogy simplifies the equilibrium characterization of the unobserved case. If there exists an equilibrium with non-decreasing education levels for the auxiliary game, then one can construct an equilibrium for the unobserved case based on that. Specifically, assign the

[9] See, for example, Johnson (2006) and Rojstaczer and Healy (2010).

[10] Alternatively, Boleslavsky and Cotton (2015) shows that coarse grading can improve social welfare by enhancing schools' investments in education quality when schools compete in placing graduates.

auxiliary game's equilibrium outcome to $\{z^u(\theta), W^u(z)\}$. We conclude that $z^u(\theta)$ solves the school's problem given $W^u(z)$, as it maximizes $MP^u(z,\theta)$ pointwise and is non-decreasing. Moreover, $W^u(z)$ is derived from the correct market belief over $z^u(\theta)$. Thus, $z^u(\theta)$ and $W^u(z)$ satisfy the equilibrium conditions of the unobserved case. As $z^u(\theta)$ has also determined the cutoff type θ_0^u, the tuition scheme $T^u(z)$ can be derived analogously to the observed case.[11] This closes the equilibrium characterization of the unobserved case.

In the following, we instead study the auxiliary game and focus on the school-optimal separating equilibrium. Given Assumption 1, we have $G_{z\theta}(z,\theta) < 0$ if $z > 0$, and thus, the single-crossing property holds. This condition means that it is less costly for the school to serve a higher-ability worker. The next proposition shows that the school-optimal separating equilibrium exists in the unobserved case.

Proposition 3. *The school-optimal separating equilibrium exists, such that*

(i) $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$*;* $z^u(\theta)$ *satisfies the first-order condition*

$$Q_z(z^u(\theta),\theta) + Q_\theta(z^u(\theta),\theta) \cdot \theta^{u\prime}(z^u(\theta)) - G_z(z^u(\theta),\theta) = 0, \tag{4.1}$$

where $\theta^u(z)$ *is the inverse function of* $z^u(\theta)$*, being differentiable on* $[\theta_0^u, \bar{\theta}]$*.*

(ii) $z^u(\theta)$ *is increasing over* $[\theta_0^u, \bar{\theta}]$*, and thus,* $W^u(z^u(\theta)) = Q(z^u(\theta),\theta)$ *for all* $\theta \in [\theta_0^u, \bar{\theta}]$*.*

Proposition 3 characterizes the equilibrium education function $z^u(\theta)$. It indicates that the cutoff type and his education level coincide for both the observed and unobserved case. In Appendix A.1, we show that if there is no exclusion in the observed case (i.e., $\theta_0^o = \underline{\theta}$), then the unobserved case has a unique separating equilibrium outcome, which is given above; otherwise (i.e., $\theta_0^o > \underline{\theta}$) there exists a continuum of separating equilibrium outcomes, and in each of them, we have $\theta_0^u \geq \theta_0^o$ and $z^u(\theta_0^u) \geq z^o(\theta_0^o)$. In addition, it is shown in Appendix A.2 that the school-optimal separating equilibrium is also the least-cost separating equilibrium in the sense that the cutoff type chooses his full-information optimal education level under the total cost function $C(z,\theta) + T^u(z)$.

The next theorem presents the paper's main result. In contrast with the observed case, the worker chooses more education in the unobserved case. In particular, a worker who has a higher ability than the cutoff type chooses strictly more education in the unobserved case.

[11] Unlike the observed case, it entails some loss of generality to assume that tuition is exorbitantly high for the off-path education, as the school cannot influence the market's belief over the tuition scheme. However, it is natural to smoothly extend T^u to $\mathbb{R}_+$; we show in the Appendix that dosing so is incentive compatible.

Theorem 1. *In contrast with the observed case, the worker chooses more education in the unobserved case. Specifically,* $z^u(\theta) \geq z^o(\theta)$ *on* $[\underline{\theta}, \bar{\theta}]$*, with strict inequality for* $\theta > \theta_0^u$*.*

Proof. Given Assumption 1, $MP^o(z, \theta)$ is strictly quasiconcave in z. Because $z^o(\theta)$ is the unique maximizer of $MP^o(z, \theta)$, it suffices to prove that $MP_z^o(z^u(\theta), \theta) \leq 0$, with strict inequality for $\theta > \theta_0^u$. This is given by the following:

$$\begin{aligned} MP_z^o(z^u(\theta), \theta) &= S_z(z^u(\theta), \theta) + \frac{1 - F(\theta)}{f(\theta)} C_{z\theta}(z^u(\theta), \theta) \\ &= Q_z(z^u(\theta), \theta) - G_z(z^u(\theta), \theta) \\ &\leq Q_z(z^u(\theta), \theta) + Q_\theta(z^u(\theta), \theta) \cdot \theta^{u\prime}(z^u(\theta)) - G_z(z^u(\theta), \theta) \\ &= 0. \end{aligned}$$

The second equality is given by the definition of $G(z, \theta)$; the inequality results from the monotonicity of $z^u(\theta)$ on $[\theta_0^u, \bar{\theta}]$; the last equality is due to (4.1). Furthermore, for $\theta > \theta_0^u$, the second term in (4.1) is positive, and thus, the above inequality becomes strict. □

As immediately implied by Theorem 1, Corollary 2 below shows that the school obtains a lower equilibrium payoff in the unobserved case than in the observed case.

Corollary 2. *In the unobserved case, the school's expected profit* Π^u *is strictly lower than its expected profit* Π^o *in the observed case.*

Proof. From Proposition 3, we have $MP^u(z^u(\theta), \theta) = MP^o(z^u(\theta), \theta)$. Because $z^o(\theta)$ is the unique maximizer of $MP^o(z, \theta)$ and $z^u(\theta) > z^o(\theta)$ for $\theta > \theta_0^u = \theta_0^o$, we have

$$\Pi^o - \Pi^u = \int_{\theta_0^o}^{\bar{\theta}} [MP^o(z^o(\theta), \theta) - MP^o(z^u(\theta), \theta)] dF(\theta) > 0.$$

Thus, the school is worse off in the unobserved case. □

The difference between the observed and unobserved case is driven by a signal jamming effect. The worker's signal is "jammed" in the unobserved case since the labor market does not observe the actual cost of education. Specifically, the labor market cannot distinguish the impact of a change in tuition from that of cost heterogeneity on the change in education. To illustrate, suppose that the school lowers tuition so that the worker chooses more education than in the initial state. When the labor market observes the tuition change, it cuts wages, as any education level now corresponds to a lower-ability worker. In contrast, when the labor market does not observe the tuition change, it does not adjust wages despite that tuition changes; thus, the worker is willing to pay more for additional education. Conversely, if the school raises tuition such that education decreases, then the labor market will *raise* wages in the observed case; thus, the worker's willingness to pay is lower in the unobserved case. This reveals that the worker is more sensitive to tuition changes in the unobserved case.

From the school's perspective, the demand is more elastic in the unobserved case. Note that the LHS of (4.1) represents the marginal profit of education in the unobserved case; the second term represents the signal jamming effect and is positive. In comparison, in the observed case, rearranging the first-order condition of $MP^o(z,\theta)$, we have

$$MP_z^o(z,\theta) = Q_z(z,\theta) - G_z(z,\theta).$$

Thus, the school's marginal profit is higher in the unobserved case than in the observed case. This provides the school with an incentive to "fool" the labor market with secret price cuts; that is, the school secretly supplies more education and persuades the labor market that the worker is more productive than is actually the case. In equilibrium, the labor market correctly anticipates the school's incentive and offers lower wages, as education is inflated. This reduces the worker's willingness to pay, and thus, the school achieves lower profits.

4.1 Implications for Tuition Transparency

We have shown that tuition cuts lead to smaller increases in demand in the observed case than in the unobserved case. This is because when the tuition cuts are publicly observed, the increase in demand is mitigated by the cheaper tuition reducing the signaling value of education. Hence, tuition cuts are less profitable in the observed case. Here, we show further that tuition is always more expensive in the observed case. Specifically, the tuition scheme in the unobserved case is uniformly lower than that in the observed case over the common domain of education. This is illustrated in Panel (a) of Figure 2.

Proposition 4. *$T^u(z) \leq T^o(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$, with strict inequality for $z > z^o(\theta_0^o)$.*

Proof. From the worker's first-order condition in both cases, we have

$$\frac{d}{dz}[W^o(z) - T^o(z)] = C_z(z,\theta^o(z)) \quad \text{and} \quad \frac{d}{dz}[W^u(z) - T^u(z)] = C_z(z,\theta^u(z)).$$

According to Theorem 1, $z^o(\theta) \leq z^u(\theta)$ on $[\theta_0^o, \bar{\theta}]$. As both $z^o(\theta)$ and $z^u(\theta)$ are increasing, $\theta^o(z) \geq \theta^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. Hence, $C_z(z,\theta^o(z)) \leq C_z(z,\theta^u(z))$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. This implies that $W^o(z) - T^o(z) \leq W^u(z) - T^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. Since $W^o(z) = Q(z,\theta^o(z))$ and $W^u(z) = Q(z,\theta^u(z))$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$, $W^o(z) \geq W^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. Hence, it is readily confirmed that $T^o(z) \geq T^u(z)$ on $[z^o(\theta_0^o), z^o(\bar{\theta})]$. $\square$

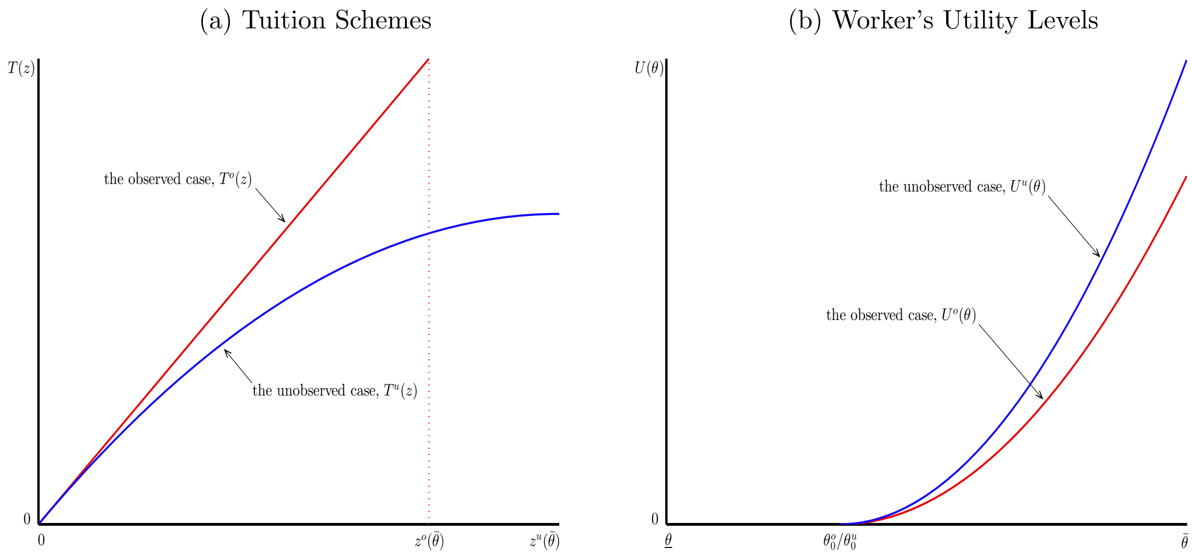

Figure 2: **Implications for Tuition Transparency.** This figure compares tuition rates and the worker's utility level between the observed and unobserved case. This figure considers the same numerical example as Figure 1, such that (a) $T^o(z) = \frac{z}{3}$ and $T^u(z) = -\frac{z^2}{4} + \frac{z}{3}$; (b) $U^o(\theta) = \frac{3}{4}(\theta - \frac{1}{3})^2$ and $U^u(\theta) = (\theta - \frac{1}{3})^2$.

Furthermore, from the worker's first-order condition in the unobserved case, we have

$$T^{u\prime}(z) = W^{u\prime}(z) - C_z(z, \theta^u(z)).$$

Substituting this equation into (4.1), and noticing that $W^u(z) = Q(z, \theta^u(z))$, we obtain

$$T^{u\prime}(z) = \frac{1 - F(\theta^u(z))}{f(\theta^u(z))} \left[-C_{z\theta}(z, \theta^u(z)) \right]. \tag{4.2}$$

Equation (4.2) states that in the unobserved case, the marginal tuition equals the marginal information rent extracted by the worker. In contrast to the observed case, as indicated by the comparison between (4.2) and (3.3), the optimal tuition scheme in the unobserved case does not undo the signaling effect. The reason is that the loss in the social surplus caused by over-education will be compensated by the labor market overpaying the worker, as the labor market will overestimate the worker's ability if the school secretly cuts tuition. In addition, (4.2) states that the marginal tuition vanishes at the highest education level. This implies that the school offers quantity discounts (i.e., $T(z)/z$ is declining) for higher education levels in the unobserved case. This echoes the classic screening model of Maskin and Riley (1984), in which quantity discounts are also optimal at the right tail of the distribution.

In terms of the worker's payoff, note that in both cases, the market belief about tuition is correct in equilibrium; thus, given the equilibrium tuition scheme, the continuation game is indeed Spence's signaling game as if the worker's cost function was given by the total cost.

Because the tuition scheme is uniformly lower in the unobserved case, the signaling costs are lower in this case. Consequently, the worker has a higher utility level in the unobserved case than in the observed case. This is illustrated in Panel (b) of Figure 2. Formally, we have

Proposition 5. *$U^u(\theta) \geq U^o(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, with strict inequality for $\theta > \theta_0^o$.*

Proof. For $\theta \in (\theta_0^o, \bar{\theta}]$, by Lemma 1 and Theorem 1, we have

$$U^u(\theta) - U^o(\theta) = \int_{\theta_0^o}^{\theta} [C_\theta(z^o(s), s) - C_\theta(z^u(s), s)] ds > 0.$$

The inequality is due to $C_{z\theta} < 0$ and $z^o(\theta) < z^u(\theta)$. For $\theta \in [\underline{\theta}, \theta_0^o]$, $U^u(\theta) = U^o(\theta) = 0$. □

Propositions 4 and 5 imply that policies that improve the transparency of net prices at colleges and universities through mandatory disclosure may unintentionally induce more expensive education and harm students. These policies, such as U.S. Code § 1015a, require schools to publicly disclose their net prices, which are usually not previously observed by employers. On the one hand, this reduces the search costs of students, thereby stimulating the competition between schools and lowering prices; on the other hand, this also allows schools to commit to high prices and not dilute the signaling value of a high-cost education by means of fee waivers, financial aid and so forth. It is thus possible that such policies ultimately raise education costs and harm students. Hence, policymakers should not overlook the potential drawbacks of these mandatory disclosure policies.

4.2 Education Comparison

We have shown that in the unobserved case, the worker chooses more education than in the observed case. For completeness, we compare the education function in the unobserved case with other benchmarks in this paper. The next proposition states that the education levels in the unobserved case are bounded above by that of Spence's signaling game. This result is illustrated in Figure 3.

Proposition 6. *In the unobserved case, the worker chooses strictly less education than in Spence's signaling game, that is, $z^u(\theta) < z^s(\theta)$ on $[\underline{\theta}, \bar{\theta}]$.*

The intuition is clear, as the unobserved case is essentially Spence's signaling game with higher costs, meaning that it yields lower education levels than Spence's model.

To see how $z^u(\theta)$ differs from $z^{fb}(\theta)$, as a reference point, note that $z^o(\theta_0^o) < z^{fb}(\theta_0^o)$ and $z^o(\bar{\theta}) = z^{fb}(\bar{\theta})$. Since $z^u(\theta) \geq z^o(\theta)$, holding strictly for $\theta > \theta_0^o$, continuity implies that

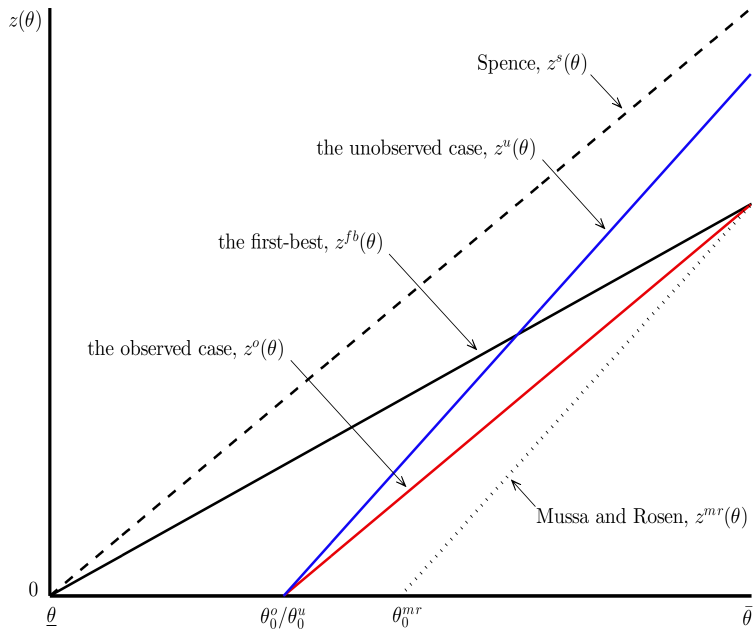

Figure 3: **All Education Functions.** This figure illustrates all the equilibrium education functions that we have discussed in this paper. This figure considers the same numerical example as Figure 1, such that $z^u(\theta) = 2\theta - \frac{2}{3}$, and recall that $z^{fb}(\theta) = \theta$, $z^s(\theta) = \frac{3}{2}\theta$, $z^o(\theta) = \frac{3\theta-1}{2}$ and $z^{mr}(\theta) = 2\theta - 1$.

$z^u(\theta)$ intersects $z^{fb}(\theta)$ from below at least once. Moreover, under some mild conditions—the following Assumption 2, for example—we show that $z^u(\theta)$ is single-crossing $z^{fb}(\theta)$, i.e., there exists a unique cutoff type such that all lower types obtain less education than the first-best while the others obtain more than the first-best (see Figure 3).

Assumption 2. *The function*

$$Q_\theta(z^{fb}(\theta), \theta) \cdot \theta^{fb'}(z) + \frac{1 - F(\theta)}{f(\theta)} C_{z\theta}(z^{fb}(\theta), \theta) \tag{$*$}$$

is single-crossing in θ.[12]

Proposition 7. *Given Assumption 2, there exists a unique cutoff type $\theta^w \in (\theta^*, \bar{\theta})$ such that $z^u(\theta) < z^{fb}(\theta)$ on $[\underline{\theta}, \theta^w)$ and $z^u(\theta) > z^{fb}(\theta)$ on $(\theta^w, \bar{\theta}]$, where $\theta^* > \theta_0^u$ is the root of $(*)$.*

In the unobserved case, there are two competing forces that pull the education function away from the first-best benchmark. On the one hand, the signal jamming effect provides the school with an incentive to supply more education. On the other hand, more education means more information rents to the worker. Since the cost of information rents ultimately vanishes as type approaches the top, the school unambiguously over-supplies education on

[12] A function $g(x)$ is single-crossing in x if given some x^*, $g(x) < 0$ for $x < x^*$ and $g(x) > 0$ for $x > x^*$.

some upper interval of the spectrum. Assumption 2 ensures that the relative significance of the two forces alters only once, thus it rules out the possibility of multiple intersections between $z^u(\theta)$ and $z^{fb}(\theta)$. Proposition 7 reveals that under-education is slighter on a lower interval of the spectrum in the unobserved case than in the observed case; it also provides a lower bound for the length of this interval. However, since over-education also occurs in the unobserved case, whether the observed or unobserved case yields higher social welfare remains ambiguous. In the next section, we examine under what circumstances either case yields higher social welfare than the other.

In this paper, we performed a pairwise comparison of different education functions. When schools are competitive, signaling leads to over-education, i.e., $z^s(\theta) > z^{fb}(\theta)$. In contrast, when schools have market power, the equilibrium education functions vary with the labor market's information. Specifically, the table below summarizes the correspondence between the equilibrium education function and information structure.

Table 1: **Education Functions under Different Information Structures.**

		Labor Market Observes Tuition	
		No	Yes
Labor Market Observes Type	No	$z^u(\theta)$	$z^o(\theta)$
	Yes	$z^{mr}(\theta)$	

As illustrated by Table 1, when the labor market observes the worker's ability and the school's tuition scheme, the model is Mussa and Rosen's screening game. A higher-ability worker benefits from his productivity and cost advantage over others. To incentivize truth-telling, the school leaves information rents to the worker and thus under-supplies education, that is, $z^{mr}(\theta) < z^{fb}(\theta)$. When the labor market observes only the tuition scheme, a higher-ability worker cannot benefit directly from his productivity advantage, and thus, signaling arises. Signaling mitigates the screening distortion since the school incurs lower information rents that stem from worker cost heterogeneity only. Thus, $z^{mr}(\theta) < z^o(\theta) < z^{fb}(\theta)$. Finally, when the labor market observes neither the tuition scheme nor the worker's ability, the worker becomes more sensitive to tuition changes, and thus, the demand for education is more elastic than in the observed case. This makes price cuts relatively more profitable for the school and induces it to supply more education; therefore, $z^o(\theta) < z^u(\theta)$.

5 Signaling Intensity and Welfare

In Section 3, we argued that signaling can mitigate the downward distortion due to screening. Then, will signaling being more intense result in more distortion cuts and thus higher social welfare in the observed case? Moreover, note that signaling induces over-education which occurs in the unobserved case, if signaling is sufficiently intense, will the observed case yield higher social welfare than the unobserved case?

To conduct such comparative statics, we shall first define the intensity of signaling. To facilitate the analysis, we parameterize the general model in the following.

Parametric setting. Assume that $Q(z,\theta) = \gamma\theta z + z$ with $\gamma > 0$, $C(z,\theta) = z^2 + z - \theta z$, and $\theta \sim U[0,1]$. Applying the previous results, we have $z^{fb}(\theta) = \frac{(\gamma+1)\theta}{2}$, $z^s(\theta) = \frac{(2\gamma+1)\theta}{2}$, $z^o(\theta) = \frac{(\gamma+2)\theta-1}{2}$ and $z^u(\theta) = (\gamma+1)\theta - \frac{\gamma+1}{\gamma+2}$. Define the intensity of signaling to be the ratio of the over-invested education in Spence's model, i.e., $z^s(\theta) - z^{fb}(\theta)$, to the first-best education level $z^{fb}(\theta)$ for $\theta > 0$. Substituting, we have

$$\frac{z^s(\theta) - z^{fb}(\theta)}{z^{fb}(\theta)} = \frac{\gamma}{\gamma+1}.$$

Clearly, the intensity of signaling is increasing in the parameter γ. To see the idea, note that the larger γ is, the stronger complementarity between the worker's ability and education. In Spence's model, higher education induces the labor market to regard the worker as having higher ability; thus, if ability complements education to a larger extent, the marginal benefit of education will be higher, thereby enhancing signaling through education.

Then, we examine how the signaling intensity affects signaling mitigating the screening distortion in the observed case. Similarly, we define the extent of the downward distortion in the observed case as the ratio of the under-supplied education, i.e., $z^{fb}(\theta) - z^o(\theta)$, to the first-best education level $z^{fb}(\theta)$ for $\theta > 0$. Substituting, we have

$$\frac{z^{fb}(\theta) - z^o(\theta)}{z^{fb}(\theta)} = \frac{1-\theta}{(\gamma+1)\theta}.$$

For any fixed $\theta \in (0,1)$, the extent of the downward distortion is decreasing in γ. This means that the more intense signaling is, the more screening distortion is mitigated.

In the unobserved case, however, the more intense signaling is, the larger over-education. Specifically, simple calculation yields that $\theta^w = \frac{2}{\gamma+2}$, which is decreasing in γ. This implies that the over-education region is increasing in the intensity of signaling. Subtracting the total surplus of the unobserved case from that of the observed case, we have

$$\int_{\theta_0^o}^{\bar{\theta}} \left[S(z^o(\theta),\theta) - S(z^u(\theta),\theta)\right] dF(\theta) = \frac{\gamma(\gamma-1)(\gamma+1)^3}{12(\gamma+2)^3}.$$

Clearly, the RHS is positive if and only if $\gamma > 1$; that is, if signaling is sufficiently intense, then the observed case yields higher social welfare than the unobserved case, as then over-education will be a relatively serious issue in the unobserved case. Intuitively, if signaling becomes more tempting, the school finds it more profitable to secretly supply more education to improve the labor market's perception of the worker.

Furthermore, recall that signaling exerts opposite welfare effects between the Spencian world and the observed case. Comparing total surplus between the two cases, we have

$$\int_{\underline{\theta}}^{\bar{\theta}} [S(z^o(\theta), \theta) - S(z^s(\theta), \theta)]\, dF(\theta) = \frac{(\gamma^2 + \gamma - 1)(\gamma^2 + 3\gamma + 1)}{12(\gamma + 2)^2}.$$

It is clear that the observed case yields higher social welfare if and only if $\gamma > \frac{\sqrt{5}-1}{2}$.[13]

This finding has welfare implications for the market structure of signals (which refer to education here). Note that when the market is served by perfectly competitive sellers of signals, the equilibrium outcome is predicted by Spence's model; when the market is served by a monopoly, the equilibrium outcome is predicted by the current model. Therefore, when the buyer's signaling incentive is sufficiently strong, monopoly can yield higher social welfare than a perfectly competitive market. This implies that introducing competition among signal sellers is not necessarily socially beneficial.

6 Discussion

In this section, we first discuss how to apply our model to other vertical relationships in which signaling prevails, such as conspicuous consumption and advertising. Then, we discuss how the main result will be affected if we change some of our modelling assumptions.

6.1 Applications of the Model

We first consider the application of conspicuous consumption. Parallel to the case of job market signaling, a retailer chooses a price schedule $T(z)$ for a luxury good, where z stands for the quality of the good. Then, à la Bagwell and Bernheim (1996), a consumer chooses the quality of the good he will purchase to signal his unobserved wealth (*type*) θ to the

[13] Analogously, the unobserved case yields higher social welfare than the Spencian world if and only if γ is larger than some cutoff less than $\frac{\sqrt{5}-1}{2}$. Thus, the domain of γ can be partitioned into four divisions in which the three cases rank differently in terms of social welfare. It follows that as the intensity of signaling rises (i.e., γ increases), the case that yields the highest social welfare will be, respectively, Spence's model, the unobserved case and the observed case.

social contacts, who observe z and form some belief about θ accordingly. In the spirit of the seminal work of Veblen (1899), the social contacts reward the consumer based on z. The reward scheme $W(z)$ is given by the social contacts' expected benefit $\mathbb{E}[Q(z, \theta)]$ from the consumer. The benefit function $Q(z, \theta)$ is increasing in both arguments. This is because a social contact benefits more from establishing relationship with wealthier people, and from interacting with people who consume higher quality goods (e.g., the goods might be non-exclusive or exhibit positive externalities). Moreover, the consumer derives intrinsic utility from the luxury good. The intrinsic utility function $V(z, \theta)$ is increasing in the quality z. More important, the single-crossing property holds: $V_{z\theta} > 0$. This condition captures the feature that a wealthier individual has higher marginal utility from consuming a luxury good. For example, a buyer of a yacht can voyage more often if he is richer, as he is better able to afford the fuel costs and maintenance fees. Thus, given the price schedule T and the reward scheme W, a type-θ consumer who chooses quality z has utility

$$U(z, \theta) := W(z) + V(z, \theta) - T(z).$$

The retailer's profit is simply given by the revenue $T(z)$ minus the cost $C(z)$. As a result of such parallel payoff structures, we can smoothly apply the analysis in the case of job market signaling to the current environment.

We now turn to the application of advertising. In this case, a media company chooses a price schedule $T(z)$ for advertising messages, where z denotes advertising level. Then, à la Milgrom and Roberts (1986), a producer that has just developed a new product chooses its advertising level to signal the unobserved quality (*type*) θ of the product to consumers, who observe z and form some belief about θ. The producer's revenue has two sources: the purchase in the introductory stage and the repeat purchase in the post-introductory stage. In the introductory stage, $D(z) \geq 0$ consumers become aware of the product and each purchases one unit at a price equal to the expected quality $\mathbb{E}(\theta)$. The demand function $D(z)$ is increasing in z, as more advertising results in higher consumer awareness. Then, in the post-introductory stage, the product's actual quality θ is revealed, and thus, the consumers are willing to purchase the good again at a price equal to θ. We assume that the consumers who were unaware of the product do not purchase the good in the post-introductory stage. Thus, the producer's total revenue is $D(z)[\mathbb{E}(\theta) + \theta]$. Given the price schedule T and assume zero production cost, a type-θ producer who chooses advertising level z has a net payoff

$$U(z, \theta) := D(z)[\mathbb{E}(\theta) + \theta] - T(z).$$

Clearly, the single-crossing property holds: $U_{z\theta} > 0$. This is due to the complementarity between advertising and quality: the marginal revenue of the introductory advertising is higher if the product is of higher quality thereby allowing the producer to charge a higher price in the post-introductory stage. The media company's profit equals the revenue $T(z)$ minus the production cost $C(z)$. As such, the payoff structure is also parallel to that of job market signaling, and thus, the analysis in this paper can be applied analogously.

6.2 Extensions of the Model

To isolate the impacts of pricing transparency on the degree of signaling and welfare, we assumed in this paper that the seller of signals is a monopolist and the buyer has a type-independent participation constraint. In a working paper, Lu (2018), we further investigate how (horizontal) competition affects the seller's pricing strategy and the degree of signaling for both the observed and unobserved case. In contrast with the current model, the buyer's preference is two-dimensional: the vertical preference parameter is in conformity with the current model; the horizontal preference parameter captures the buyer's outside option; these parameters are independent and both privately known. Hence, the buyer has now a type-dependent participation constraint.

We find that in contrast to the observed case, the allocation of signals is more dispersed in the unobserved case. Specifically, an interval of higher types obtain higher levels of signal than in the observed case, with the highest types obtaining higher levels than the first-best, whereas those lower types obtain lower levels of signal than in the observed case. We show that the length of such an lower interval, in which the buyer purchase more signals in the observed case, is increasing in the degree of horizontal differentiation and vanishes as the degree approaches zero. This means that there is no discontinuity in the current paper's results when we disturb the participation constraint somewhat.

In addition, the current paper's results still hold if we change nonlinear tuition to linear tuition or change continuous types to discrete types. A somewhat special case is linear tuition with discrete types. Without loss of generality, suppose that there are only two types, low and high, and the school chooses a uniform tuition rate. In the observed case, the least-cost separating equilibrium exists, in which the high type obtains more education than the low type, and the latter is indifferent between revealing own type and imitating the former. In the unobserved case, however, such an equilibrium does not exist because the high education level is so high that the low type strictly prefers to reveal his type. The intuition is similar: the high education level must be relatively too high for the low type to imitate the high type, such that the school finds it unprofitable to cut the price and gain market share.

A non-robust phenomenon emerges due to the school's inability to price discriminate. That is, when the high type strictly prefers to separate himself from the low type, the school has an incentive to squeeze him by raising tuition. The reason is that the high type is less sensitive to tuition changes, as a decrease in education will cause him to be regarded as the low type even if this decrease is due to higher tuition. Therefore, the school faces a trade-off between squeezing the high type and maintaining the low type's market share. If the gap between the two types and the fraction of the high type are large enough, squeezing the high type is more profitable, such that in equilibrium the low type is excluded from education and the high type is indifferent between choosing the equilibrium high education level and deviating downward optimally.

7 Conclusion

In this paper, we developed classic signaling models by allowing a strategic player to affect signaling cost. A seller chooses a price schedule for a product, and a buyer with a hidden type chooses how much to purchase as a signal to receivers. The equilibrium depends critically on whether receivers observe the price schedule. In the observed case, the seller internalizes signaling in screening, causing a downward distortion. In the unobserved case, the buyer is more sensitive to price changes. This leads to a more elastic demand for signals and induces the seller to lower prices. In equilibrium, the buyer chooses a higher quantity and obtains higher utility than in the observed case, whereas the seller achieves lower profits than in the observed case. We show that price transparency can improve social welfare when the buyer's signaling incentive is relatively strong. Such a framework can be applied to schools choosing tuition, retailers selling luxury goods and media companies selling advertising messages.

A Appendix

A.1 Omitted Proofs

Proof of Proposition 3.

Proof. We first prove that a separating equilibrium exists in the unobserved case. Fix some admissible initial point $(\theta_0^u, z^u(\theta_0^u))$. Let $MP(\theta, \hat{\theta}, z)$ be type-θ's marginal profit if he obtains education level z and is believed by the labor market as type-$\hat{\theta}$. In particular, $MP(\theta, \theta, z)$ equals $MP^o(z, \theta)$ which is regular by Assumption 1. Moreover, $MP_2(\theta, \hat{\theta}, z) = Q_\theta(z, \hat{\theta}) > 0$, $MP_{13}(\theta, \hat{\theta}, z) = -G_{z\theta}(z, \theta) > 0$, and $MP_3(\theta, \hat{\theta}, z)/MP_2(\theta, \hat{\theta}, z)$ is increasing in θ. Applying Mailath (1987, Theorem 3), we have that there exists a unique separating equilibrium given the initial condition, such that the equilibrium education function $z^u(\theta)$ satisfies (4.1) and is increasing over the interval $[\theta_0^u, \bar{\theta}]$.

To find the school-optimal separating equilibrium, it suffices to pin down the initial point. We consider two cases. First, $\theta_0^o = \underline{\theta}$. Note that the lowest possible wage for any education level $z > 0$ is $Q(z, \underline{\theta})$. Thus, for every pair (z, θ) with $z > 0$, we have

$$MP^u(z, \theta) \geq Q(z, \underline{\theta}) - G(z, \theta) \geq Q(z, \underline{\theta}) - G(z, \underline{\theta}) = MP^o(z, \underline{\theta}).$$

The second inequality is due to $G_{z\theta} < 0$ if $z > 0$. Since $\theta_0^o = \underline{\theta}$, $MP^u(z^o(\underline{\theta}), \underline{\theta}) \geq 0$; that is, the marginal profit of the lowest type can be at least non-negative. Thus, $\theta_0^u = \underline{\theta}$. Note too that $MP^u(z^u(\theta), \theta) = MP^o(z^u(\theta), \theta)$, as types reveal in equilibrium. Then, it is optimal for the school to choose $z^u(\underline{\theta}) = z^o(\underline{\theta})$, because the labor market cannot punish this choice by holding a worse belief than $\underline{\theta}$ and $z^o(\underline{\theta})$ maximizes $MP^o(z, \underline{\theta})$ by definition. Thus, if $\theta_0^o = \underline{\theta}$, then the separating equilibrium outcome is unique such that $(\theta_0^u, z^u(\theta_0^u)) = (\underline{\theta}, z^o(\underline{\theta}))$.

Second, $\theta_0^o > \underline{\theta}$. In this case, $(\theta_0^u, z^u(\theta_0^u))$ and thus the equilibrium outcome is not unique. From Mailath (1987, Theorem 3), for every separating equilibrium, $z^u(\theta)$ satisfies (4.1) and is increasing. Analogously to the proof of Theorem 1, we have $z^u(\theta) \geq z^o(\theta)$ on $[\theta_0^u, \bar{\theta}]$ with strict inequality for $\theta > \theta_0^u$. This implies that $MP^u(z^u(\theta), \theta) \leq MP^o(z^o(\theta), \theta)$, as $z^o(\theta)$ is the unique maximizer of $MP^o(z, \theta)$, and $MP^u(z^u(\theta), \theta) = MP^o(z^u(\theta), \theta)$. By the definition of the cutoff type, we have $\theta_0^u \geq \theta_0^o$ in every separating equilibrium of the unobserved case. Thus, we have determined the lower bound of $(\theta_0^u, z^u(\theta_0^u))$. In Appendix A.2, we show that the school-optimal separating equilibrium exists in this case such that $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$. In summary, in both cases, $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$; thus, Proposition 3 is proven. □

Proof of Proposition 6.

Proof. We only need to prove that $z^u(\theta) < z^s(\theta)$ on $[\theta_0^u, \bar{\theta}]$. From the first-order conditions, we can derive $W^u(z)$ and $W^s(z)$, respectively, by the IVPs below:

$$W^{u\prime}(z) = G_z(z, \theta^u(w,z)) \ \text{ and } \ W^{s\prime}(z) = C_z(z, \theta^s(w,z)),$$

with the initial points $(z^o(\theta_0^o), W^u(z^o(\theta_0^o)))$ and $(z^o(\theta_0^o), W^s(z^o(\theta_0^o)))$ for $W^u(z)$ and $W^s(z)$, respectively. It is easy to see that $G_z(z,w) \geq C_z(z,w)$ in any common domain of (z,w). From Corollary 1, we have $z^o(\theta) < z^s(\theta)$ on $[\underline{\theta}, \bar{\theta}]$, and thus, $\theta_0^o > \theta^s(z^o(\theta_0^o))$. This implies that $W^u(z^o(\theta_0^o)) > W^s(z^o(\theta_0^o))$. Then, appealing to Hartman (1964, Corollary 4.2, page 27), we have $W^u(z) > W^s(z)$ in any common domain. This implies that $\theta^u(z) > \theta^s(z)$ in any common domain; therefore, $z^u(\theta) < z^s(\theta)$ on $[\theta_0^u, \bar{\theta}]$. □

Proof of Proposition 7.

Proof. We only need to study the interval $(\theta_0^u, \bar{\theta})$. We have shown that $z^u(\theta)$ intersects $z^{fb}(\theta)$ from below at least once. Note that $z^u(\theta_0^u) = z^o(\theta_0^u) < z^{fb}(\theta_0^u)$ and $z^u(\bar{\theta}) > z^o(\bar{\theta}) = z^{fb}(\bar{\theta})$. If there are multiple intersections, then $z^u(\theta)$ intersects $z^{fb}(\theta)$ at least three times. Denote by $W^{fb}(z)$ the wage schedule in the first-best benchmark. Since both $W^u(z)$ and $W^{fb}(z)$ are increasing, it suffices to prove that $W^u(z)$ intersects $W^{fb}(z)$ only once. Suppose that $z^u(\theta)$ intersects $z^{fb}(\theta)$ at some θ^w, then $W^u(z)$ intersects $W^{fb}(z)$ at $z^{fb}(\theta^w)$. Differentiating both $W^u(z)$ and $W^{fb}(z)$ at $z^{fb}(\theta^w)$, respectively, we have

$$\begin{aligned} W^{u\prime}(z^{fb}(\theta^w)) &= C_z(z^{fb}(\theta^w), \theta^w) - \frac{1-F(\theta^w)}{f(\theta^w)} C_{z\theta}(z^{fb}(\theta^w), \theta^w), \\ W^{fb\prime}(z^{fb}(\theta^w)) &= Q_z(z^{fb}(\theta^w), \theta^w) + Q_\theta(z^{fb}(\theta^w), \theta^w) \cdot \theta^{fb\prime}(z^{fb}(\theta^w)). \end{aligned}$$

The first equation results from the first-order condition of $MP^u(z,\theta)$; the second is just the total derivative of $W^{fb}(z)$. Rearranging and substituting (2.1), we have

$$W^{fb\prime}(z^{fb}(\theta^w)) - W^{u\prime}(z^{fb}(\theta^w)) = Q_\theta(z^{fb}(\theta^w), \theta^w)\theta^{fb\prime}(z^{fb}(\theta^w)) + \frac{1-F(\theta^w)}{f(\theta^w)} C_{z\theta}(z^{fb}(\theta^w), \theta^w).$$

Given Assumption 2, the RHS can change its sign only once for different values of θ^w. Suppose that $z^u(\theta)$ intersects $z^{fb}(\theta)$ more than once, then the directions of the first three intersections are from below, from above, and from below; thereby, $W^u(z)$ intersects $W^{fb}(z)$ first from above, then from below, and then from above. This means that the LHS of the above equation will change its sign more than once, a contradiction. Thus, we conclude that

$z^u(\theta)$ intersects $z^{fb}(\theta)$ only once and from below. Then, $W^{fb\prime}(z^{fb}(\theta^w)) - W^{u\prime}(z^{fb}(\theta^w)) > 0$, meaning that the RHS of the above equation is positive. By the definition of θ^*, we have

$$Q_\theta(z^{fb}(\theta^*), \theta^*) \cdot \theta^{fb\prime}(z^{fb}(\theta^*)) + \frac{1 - F(\theta)}{f(\theta)} C_{z\theta}(z^{fb}(\theta^*), \theta^*) = 0.$$

It is readily confirmed by Assumption 2 that $\theta^w > \theta^*$. Thus, the proposition is proven. □

A.2 Equilibrium Selection for the Unobserved Case

Here, we discuss equilibrium selection for the unobserved case. We present two lemmas. By Lemma 2, we characterize the school-optimal separating equilibrium given that $\theta_0^o > \underline{\theta}$; by Lemma 3, we show that the school-optimal separating equilibrium is also the least-cost separating equilibrium with respect to the total cost of education.

Lemma 2. *Suppose that $\theta_0^o > \underline{\theta}$, then the school-optimal separating equilibrium exists in the unobserved case, such that $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$.*

Proof. As a first step, we show that the cutoff type's education level $z^u(\theta_0^u)$ is an increasing function of θ_0^u. From the proof of Proposition 3, we have $\theta_0^u \geq \theta_0^o > \underline{\theta}$. Thus,

$$MP^u(z^u(\theta_0^u), \theta_0^u) = MP^o(z^u(\theta_0^u), \theta_0^u) = 0.$$

Given Assumption 1, $MP^o(z, \theta)$ is regular; $z^o(\theta_0^u)$ is the unique maximizer of $MP^o(z, \theta_0^u)$. From the proof of Proposition 3, we have $z^u(\theta_0^u) \geq z^o(\theta_0^u)$ for each separating equilibrium. Then, regularity implies that $z^u(\theta_0^u)$ is the unique solution to the above equation given an admissible θ_0^u, and $z^u(\theta_0^u)$ is increasing. Thus, $z^u(\theta_0^u)$ is an increasing function of θ_0^u.

Second, we show that for any two admissible initial points $(\hat{\theta}_0^u, \hat{z}^u(\theta))$ and $(\tilde{\theta}_0^u, \tilde{z}^u(\theta))$, if $\hat{\theta}_0^u < \tilde{\theta}_0^u$, then $\hat{z}^u(\theta) < \tilde{z}^u(\theta)$ in any common domain. From Mailath (1987, Theorem 3), for every separating equilibrium, $z^u(\theta)$ satisfies (4.1). Rearranging (4.1), we have

$$z^{u\prime}(\theta) = \frac{Q_\theta(z^u(\theta), \theta)}{G_z(z^u(\theta), \theta) - Q_z(z^u(\theta), \theta)}.$$

From the first paragraph, if $\hat{\theta}_0^u < \tilde{\theta}_0^u$, then $\hat{z}^u(\hat{\theta}_0^u) < \tilde{z}^u(\tilde{\theta}_0^u)$. Appealing to Hartman (1964, Corollary 4.2, page 27), we have $\hat{z}^u(\theta) < \tilde{z}^u(\theta)$ in the common domain $[\tilde{\theta}_0^u, \bar{\theta}]$.

Third, we characterize $z^u(\theta)$ for all separating equilibria. To do so, we have to determine the domain of $z^u(\theta_0^u)$, which depends on the market belief off the equilibrium path. As have been shown, the lower bound of θ_0^u is θ_0^o, which is supportable if any off-path education is believed to be chosen by type θ_0^o. As the off-path belief gets gradually harsher, θ_0^u increases

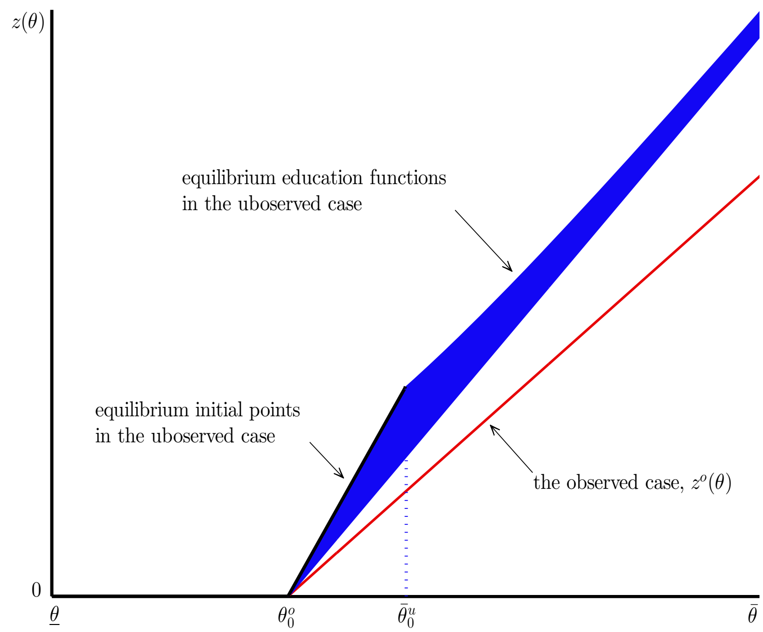

Figure 4: **The Family of Separating Equilibria.** This figure illustrates the family of separating equilibria in the unobserved case given that $\theta_0^o > \underline{\theta}$. The blue area depicts the set of equilibrium education functions. This region is uniformly above the equilibrium education function in the observed case $z^o(\theta)$. The bold line is the set of equilibrium initial points with the cutoff type ranging from θ_0^o to $\bar{\theta}_0^u$. Each point uniquely determines an equilibrium education function $z^u(\theta)$ and thus an equilibrium outcome. This figure considers the same numerical example as Figure 1, such that the set of the initial points is $\{(\theta, z)|z(\theta) = 3\theta - 1; \frac{1}{3} \leq \theta \leq \frac{1}{2}\}$.

continuously, until the labor market holds the worst belief $\underline{\theta}$ off the equilibrium path. It is without loss of generality to confine the off-path education to $[0, z^u(\theta_0^u))$ when $z^u(\theta_0^u) > 0$. Denote by $\bar{\theta}_0^u$ the upper bound of θ_0^u, which is pinned down by

$$\max_{z < z^u(\bar{\theta}_0^u)} \{Q(z, \underline{\theta}) - G(z, \bar{\theta}_0^u)\} = MP^u(z^u(\bar{\theta}_0^u), \bar{\theta}_0^u) = 0.$$

That is, the school is indifferent between allocating type-$\bar{\theta}_0^u$ the optimal off-path education such that it is believed as type-$\underline{\theta}$ and maintaining the equilibrium allocation. Therefore, we have determined the domain of $z^u(\theta_0^u)$. Then, picking any $\theta_0^u \in [\theta_0^o, \bar{\theta}_0^u]$, we can uniquely pin down a $z^u(\theta)$. Figure 4 illustrates the education functions of all separating equilibria.

Finally, we show that the initial point of the school-optimal separating equilibrium is $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$. Pick two equilibrium education functions, $\hat{z}^u(\theta)$ and $\tilde{z}^u(\theta)$, such that $\hat{z}^u(\theta) < \tilde{z}^u(\theta)$ on the common support $[\tilde{\theta}_0^u, \bar{\theta}]$. Since $z^u(\theta) \geq z^o(\theta)$ on $[\theta_0^u, \bar{\theta}]$ for every separating equilibrium, we have $\hat{z}^u(\theta) - z^o(\theta) < \tilde{z}^u(\theta) - z^o(\theta)$ on $[\tilde{\theta}_0^u, \bar{\theta}]$. Thus, regularity implies that $MP^u(\hat{z}^u(\theta),\theta) > MP^u(\tilde{z}^u(\theta),\theta)$ on $[\hat{\theta}_0^u,\bar{\theta}] \supset [\tilde{\theta}_0^u,\bar{\theta}]$. Then,

$$\Pi^u(\hat{\theta}_0^u) - \Pi^u(\tilde{\theta}_0^u) = \int_{\hat{\theta_0^u}}^{\bar{\theta}} MP^u(\hat{z}^u(\theta),\theta)dF(\theta) - \int_{\tilde{\theta_0^u}}^{\bar{\theta}} MP^u(\tilde{z}^u(\theta),\theta)dF(\theta) > 0.$$

The inequality is due to the fact that both the integrand and the integral domain of $\Pi^u(\hat{\theta}_0^u)$ are bigger than those of $\Pi^u(\tilde{\theta}_0^u)$. This result reveals that the lower the cutoff type, the higher the school's equilibrium payoff. Since $\theta_0^u \in [\theta_0^o, \bar{\theta}_0^u]$, the school-optimal separating equilibrium must be the one in which $\theta_0^u = \theta_0^o$, and thus, $z^u(\theta_0^u) = z^o(\theta_0^o)$. □

Lemma 3. *In the school-optimal separating equilibrium, the cutoff type θ_0^u chooses his full-information optimal education level under the total cost function $C(z,\theta_0^u) + T^u(z)$, i.e.,*

$$z^u(\theta_0^u) = \underset{z}{\operatorname{argmax}}\ Q(z,\theta_0^u) - C(z,\theta_0^u) - T^u(z).$$

Proof. First, we construct T^u on $\mathbb{R}_+$. On the equilibrium path, T^u is given by

$$T^u(z^u(\theta)) = S(z^u(\theta),\theta) - U^u(\theta) = S(z^u(\theta),\theta) + \int_{\theta_0^u}^{\theta} C_\theta(z^u(s),s)ds.$$

Then, we smoothly extend T^u to $\mathbb{R}_+$. First, from (4.2), we have $\lim_{z\to z^u(\bar{\theta})^-} T^{u\prime}(z) = 0$. It is thus natural to extend T^u horizontally upto $+\infty$. Second, if $z^u(\theta_0^u) > 0$, then we smoothly extend T^u to the left by extending the solution to the IVP that is defined by the differential equation in (4.2) and the initial condition that $(\theta_0^u, z^u(\theta_0^u)) = (\theta_0^o, z^o(\theta_0^o))$, until $T^u(z)$ or z reaches 0, whichever is earliest. The rest part of T^u is fixed at 0. As such, T^u is completely characterized on $\mathbb{R}_+$. To ensure that such T^u satisfies incentive compatibility, we simply assume that the labor market holds the worst belief $\underline{\theta}$ for any off-path education level, so that no type will deviate to the off-path in any case.

Thus, given T^u, it suffices to prove that the following first-order condition holds.

$$Q_z(z^u(\theta_0^u),\theta_0^u) - C_z(z^u(\theta_0^u),\theta_0^u) - T^{u\prime}(z^u(\theta_0^u)) = 0. \tag{A.1}$$

Note that $MP^o(z,\theta)$ is regular, $z^o(\theta_0^o)$ maximizes $MP^o(z,\theta_0^o)$ and $z^u(\theta_0^u) = z^o(\theta_0^o)$, thus

$$MP_z^o(z^u(\theta_0^u),\theta_0^u) = Q_z(z^u(\theta_0^u),\theta_0^u) - G_z(z^u(\theta_0^u),\theta_0^u) = 0.$$

Substituting $G_z(z^u(\theta_0^u),\theta_0^u)$ using (4.2) and the definition of $G(z,\theta)$, we obtain (A.1). This implies that the cutoff type chooses his full-information optimal education level under the total cost function. Thus, Lemma 3 is proven. □

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