Incentive Design for Agile Teams

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Motivation

An agile team is

- A cross-functional group of people with various expertise, collaborating on a high-value project on a temporary basis.
  - E.g., Amazon's "two-pizza teams", special teams in China, ...
- Flexible in designing workflow, job assignment, payments, etc.

Internal information is critical to organization design

- Employees' job attitudes respond to observed efforts of peers.
- Organization design can profoundly affect internal information.

Question

- How to design agile teams by leveraging internal information?

Overview

Holmstrom's classic remark (Holmstrom, 1982)

- What determines the choice of monitors?
- How should output be shared to provide the best incentives?
- "A big step toward understanding nonmarket organization"

We propose a novel framework of moral hazard in team

- Complementary technology + network-based peer monitoring
- Least-cost effort-inducing mechanism: workflow and rewards

Key insights

- Transparency of agents' actions reduce their incentive costs.
- Effectiveness of the transparency on incentives is diminishing.

Results

- In the optimal sequence, the agents work sequentially.
- Better connected agents work in intermediate stages.
- More important agents and sub-teams work in later stages.
- Large sub-teams are assigned to either end of the sequence.
- Simple algorithms are offered to characterize the optimum.

Implications

- Ex ante identical agents may earn different wages ex post.
- Better connected agents act as information intermediaries.
- More important agents act as monitors and earn higher wages.
- Peer-monitoring is effective but costly in providing incentives.
- Importance and size of a sub-team have different implications.

Literature

Theoretical works on incentive design with multiple agents

- Alchian & Demsetz (1972), Holmstrom (1982), Itoh (1991)
- Lazear & Rosen (1981), Gershkov et al. (2009, 2016)
- Strausz (1999), Che and Yoo (2001), Au and Chen (2019)
- Winter (2004, 2006, 2010), Zhou (2016), Halac et al. (2021)
- McAfee & McMillan (1991), Segal (1999, 2003), Bernstein & Winter (2012), Babaioff et al. (2012), Balmaceda (2016, 2018)

Empirical works on the role of internal information

- Ichino & Maggi (2000), Knez & Simester (2001), Heywood & Jirjahnm (2004), Gould & Winter (2009), Mas & Moretti (2009)
- Carpenter et al. (2009), Steiger & Zultan (2014)

Model

Model

Players and actions

- A set I of n agents manage a project owned by a principal.
- Each agent chooses whether to exert effort or not: $a \in \{0, 1\}$.
- Exerting effort is costly $(c = 1)$ while shirking is costless.

Network

- Organizational structure is given by an undirected graph g.
- Agents i and j are directly linked (*neighbors*) if $ij \in g$.
- Examples: workplace architecture, authority structure, etc.

Technology

- The project succeeds with probability $p(W)$, $W = \{i|a_i = 1\}$.
- Monotonicity: if $T \subset S$, then $p(T) < p(S)$.

Mechanism

- Principal chooses a mechanism $\{\pi, v\}$ that consists of:

1. A work sequence π s.t. agent i is the π_i-th to perform.
2. A reward scheme v s.t. agent i receives $v_i \geq 0$ upon success.

Information

- Principal observes *only* whether the project succeeds or not.
- Agent i observes j's action (i sees j) iff $ij \in g$ and $\pi_i > \pi_j$.
- Let $N_i := \{j | ij \in g, \pi_i > \pi_j\}$ be the agents whom i sees.
- Let $M_i := \{j | j \text{ sees } k_1 \text{ sees } \ldots k_r \text{ sees } i\}$ be the agents who could ultimately learn i's action if agents could communicate.

More on Technology

Importance

- Agent i is more *important* than j if for any S s.t. $i, j \in S$,

$$p(S \backslash \{i\}) \leq p(S \backslash \{j\}).$$

Complementarity

- p is *complementary* if for any S, T s.t. $T \subset S$ and for $i \notin S$,

$$p(S \cup \{i\}) - p(S) > p(T \cup \{i\}) - p(T).$$

Substitutability

- p is *substitutable* if for any S, T s.t. $T \subset S$ and for $i \notin S$,

$$p(S \cup \{i\}) - p(S) \leq p(T \cup \{i\}) - p(T).$$

Principal's Problem

Agent's payoff

- A (pure) strategy of agent i is a mapping $s_i : 2^{N_i} \to \{0, 1\}$.
- Given a strategy profile s, agent i's expected utility equals

$$U_i(s) := p(W(s))v_i - \mathbb{1}(s_i = 1),$$

where $\mathbb{1}(\cdot)$ is the indicator function.

Effort-inducing (EFI) mechanism

- A mechanism is EFI if there exists a *PBE* s.t. $W(s) = I$.
- Principal seeks the EFI mechanism with minimal total rewards.
- Denote the optimal mechanism $\{\pi^*, v^*(\pi^*)\}$.

Preliminary Analysis

Optimal Reward Scheme under Complementarity

Proposition 1.

Suppose p is complementary. For any fixed sequence π, the optimal reward scheme $v^*(\pi)$ pays agent i

$$v_i^*(\pi) = \frac{1}{p(I) - p(I \backslash (\{i\} \cup M_i))}.$$

Idea

- Agents shirk whenever they see anyone shirks.

Implications

- Agents face *implicit threats of shirking* if move sequentially.
- A more transparent action leads to a lower incentive cost.

Optimal Reward Scheme under Substitutability

Proposition 1'.

Suppose p is substitutable. For any fixed sequence π, the optimal reward scheme $v^*(\pi)$ is invariant w.r.t. π and pays agent i

$$v_i^*(\pi) = \frac{1}{p(I) - p(I\backslash\{i\})}.$$

Idea

- Agents substitute own efforts for observed shirking actions.

Implications

- Internal information has no impacts on the incentive costs.
- In what follows, assume that p is a complementary technology.

Sequential Moves and Importance Ranking

Lemma 1.
For any two agents i and j, if $ij \in g$, then $\pi_i^* \neq \pi_j^*$.

Idea

- Simultaneous moves reduce the transparency of actions.

Lemma 2.
For any two agents i and j s.t. i is more important than j and $\{k|ik \in g, k \neq j\} = \{k|jk \in g, k \neq i\}$, if in π^* either $i \in M_j^*$ or $j \in M_i^*$, then $\pi_i^* > \pi_j^*$.

Idea

- More important successors pose greater implicit threats.

Full Transparency Benchmark

Optimal Mechanism for Complete Networks

Complete network

- In a complete network, any two agents are neighbors.

Identity permutation

- Relabel the agents s.t. agent i is less important than $i+1$.

Proposition 2.

If g is a complete network and the agents are increasingly important, then the optimal mechanism $\{\pi^*, v^*\}$ satisfies that (i) π^* is the identity permutation; (ii) agent i has a reward

$$v_i^* = \frac{1}{p(I) - p(\{j|j < i\})}.$$

Incentives and Discrimination

Corollary 1.

If g is a complete network, then under the optimal mechanism $\{\pi^*, v^*\}$, v_i^* is increasing and strictly convex in i.

Idea

- Transparency of agents' actions reduce their incentive costs.
- Effectiveness of the transparency on incentives is diminishing.

Implications

- Differential rewards may occur even among identical agents.
- Similar agents may differ greatly under internal information.
- More important agents perform later and serve as monitors.
- Monitor incentivizes high efforts but is costly to incentivize.

Optimality of Complete Network

Corollary 2.

A complete network generates minimal total payoffs to the agents, and thus maximal payoff to the principal.

Idea

- Complete network yields the richest transparency.

Implications

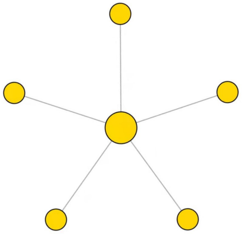
- Principal may wish to transform g into a complete network.
- "Open space" improves peer monitoring and reduces shirking.
- E.g., FB's "war room", RBS's new headquarters building, etc.

A Single Team

Star Network

Star network

- There is a unique node i s.t. every link in g involves i;
- Agent i is the *center*, and the others are *peripheral agents*.
- Examples: scientific lab, construction, book editing, etc.



A Star Network

Center's Successors are More Important Peripherals

Lemma 3.

If in π^* the center has a nonempty set of predecessors and a nonempty set of successors, then the successors are uniformly more important than the predecessors.

A sufficient statistic for the optimal sequence

- Let m be the number of center's successor(s), $0 \leq m \leq n-1$.
- It suffices to pin down m given Lemma 3.

Identity permutation

- Relabel peripheral agents by rising importance from 1 to $n-1$.
- Let center be the n-th (not necessarily the most important).

Total Rewards to Agents

According to Proposition 1

- Total rewards to agents are given by

$$V^*(m) = \underbrace{\sum_{i=1}^{n-1-m} \frac{1}{p(I) - p(\{j|j < n-m\} \backslash \{i\})}}_{\text{payoffs to the predecessors}}$$

$$+ \underbrace{\frac{1}{p(I) - p(\{j|j < n-m\})}}_{\text{payoff to the center}} + \underbrace{\sum_{i=n-m}^{n-1} \frac{1}{p(I) - p(I \backslash \{i\})}}_{\text{payoffs to the successors}}.$$

Effects of Increasing Center's Successors

Marginal benefit of increasing m

- Reducing total rewards to center and his predecessors by

$$MB(m) := \sum_{i=1}^{n-2-m} \left[\frac{1}{p(I) - p(\{j|j < n-m\} \backslash \{i\})} - \frac{1}{p(I) - p(\{j|j < n-m-1\} \backslash \{i\})} \right] + \frac{1}{p(I) - p(\{j|j < n-m\})}.$$

Marginal cost of increasing m

- Increasing total rewards to center's successors by

$$MC(m) := \frac{1}{p(I) - p(I \backslash \{n-m-1\})}.$$

Optimal Sequence

Lemma 4.
$MB(m)$ is decreasing in m, while $MC(m)$ is nondecreasing in m.

Idea

- Effectiveness of the transparency on incentives is diminishing.
- Fewer actions are observable as center has more successors.
- Each new successor is less important than the current ones.

Implications

- The optimal sequence is characterized by the integer

$$m^* := \min\{m | MB(m) \leq MC(m)\}. \tag{1}$$

- The sub-sequence of predecessors (successors) doesn't matter.

Optimal Mechanism for Star Networks

Proposition 3.

If g is a star network, then under the optimal mechanism $\{\pi^*, v^*\}$:

(i) *The center has m^* successor(s), each of them is more important than the center's predecessors, where m^* is given by (1) with $0 \leq m^* \leq n-2$;*

(ii) *v^* is given by Proposition 1 accordingly.*

(iii) *Moreover, if $\left[p(I) - p(I \backslash \{n-1\})\right] < \delta \left[p(I) - p(I \backslash \{n\})\right]$ for some small $\delta > 0$, then $m^* = 0$, where agent $n-1$ is the most important peripheral agent and agent n is the center.*

A simple algorithm to search the optimum

- Increase m from 0 in descending order of importance.
- Continue this process until $MB(m) \leq MC(m)$.

Comparative Statics: Importance of Individual Task

Parametric setting (Winter 2006)

- Let w be the number of agents who exert effort, $w \leq n$.
- The probability of success is $p(w) = \alpha^{n-w}$, $\alpha \in (0, 1)$.
- Lower α means the failure of individual task is more crucial.

Corollary 3.

In the optimal sequence π^*, the number of the center's successors $m^*(\alpha)$ is nondecreasing in α for $\alpha \in (0, 1)$.

Idea

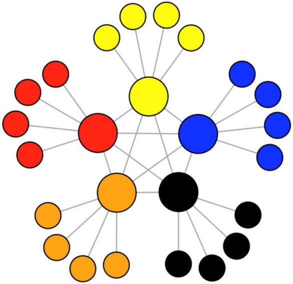
- If each task is important, agents are more willing to work.
- Implicit threat of shirking is not crucial in providing incentive.
- Thus, marginal benefit of increasing m is relatively low.

Multiple Collaborating Teams

Multi-Star Network

Multi-star network

- Multiple $(t \geq 2)$ stars with centers completely interconnected
- Examples: cross-laboratory research, modular design, etc.



A Multi-Star Network

Work Sequence cross and within Stars

Relative order of stars

- Let $\hat{n}_i \geq 3$ be the number of agents in star i, $i \in \{1, \ldots, t\}$.
- Let c_i be the center of star i and $C := \{c_i\}$ the set of centers.
- Let π^C be the sub-sequence of centers confined to C.
- Lemma 1 implies that π^C should be completely sequential.

Internal sequence of stars

- Call a peripheral agent moving before own center *predecessor*.
- Call a peripheral agent moving after own center *successor*.
- Let m_i be the number of successors of star i, $0 \leq m_i \leq \hat{n}_i - 1$.

Homogeneous Agent Importance

- First, suppose that all the agents are equally important.

Marginal benefit of increasing m_i

- Reducing the rewards to c_i and each agent j s.t. $c_i \in M_j$.
- Marginal benefit of increasing m_i is higher if c_i moves later.

Marginal cost of increasing m_i

- Increasing the total rewards to c_i's successors by

$$MC \equiv \frac{1}{p(n) - p(n-1)}.$$

Optimal Sequence with Homogeneous Importance

Lemma 5.

Suppose in π^* c_i moves before c_j, then (i) if $m_i > 0$, then $m_j = \hat{n}_j - 1$; (ii) if $m_j < \hat{n}_j - 1$, then $m_i = 0$.

Proposition 4.

The optimal sequence satisfies that for any two stars i and j such that i moves before j, (i) if $m_i = m_j = 0$, then $\hat{n}_i \geq \hat{n}_j$; (ii) if $m_i = \hat{n}_i - 1$, $m_j = \hat{n}_j - 1$, then $\hat{n}_i \leq \hat{n}_j$.

Implications

- The optimal sequence is "V-shaped" in terms of team size.
- The centers serve as the internal information intermediaries.
- Proposition 4 rules out most suboptima: only 2^t out of $t!$ left.

Simple Algorithm Given Optimal π^C

Corollary 4.

Suppose the optimal sub-sequence π^C has been determined, then π^* can be fully characterized by the following algorithm:

1. *Set each star i such that $m_i = 0$, $i \in \{1, \ldots, t\}$;*
2. *From the last star to the first star in π^C, increase m_i until*

$$MB\left(\sum m_i\right) \leq MC.$$

Two-stage algorithm for small numbers of teams

1. For each undominated π^C, apply the algorithm in Corollary 4;
2. Compare the total rewards in step 1 and select the optimum.

Bounds of the Total Number of Successors

Proposition 5.

Suppose π^* *contains totally* $\sum m_i^*$ *successors. Then,*

1. *Align the stars in ascending order of* $\hat{n}_i$, $i \in \{1, \ldots, t\}$;
2. *Apply Corollary* 4 *to this* π^C, *and let* $\bar{m}$ *be the associated total number of successors;*

1' *Align the stars in descending order of* $\hat{n}_i$, $i \in \{1, \ldots, t\}$;

2' *Apply Corollary* 4 *to this* π^C, *and let* $\underline{m}$ *be the associated total number of successors.*

It follows that $\underline{m} \leq \sum m_i^* \leq \bar{m}$.

Idea

- Increasing m_i is more profitable if actions are less transparent.
- Least action transparency when π^C is in ascending order of $\hat{n}_i$.

A Small Fraction of Agents are Monitors

Assumption 1.

For any positive integer $m \leq n-1$, we have

$$\frac{1}{m}[p(n-1) - p(n-m-1)] \geq K[p(n) - p(n-1)]$$

for some constant $K > 0$.

Proposition 6.

Given Assumption 1, the optimal sequence satisfies that $\sum m_i^*$ is bounded above by some number of order $\sqrt{n}$.

Proof sketch

- $\sum m_i^*$ is smaller than m^* of a single star with the same size n.
- m^* is bounded above by some number that is of order $\sqrt{n}$.

Homogeneous Team Size

- Now, suppose that every team has the same size $\hat{n} \geq 3$.

Relative importance of stars

- Let r_i^k be the k-th important peripheral agent of star i.
- For stars i and j, i is more important than j if

1. c_i is more important than c_j;
2. r_i^k is more important than r_j^k, for all $1 \leq k \leq \hat{n} - 1$.

- Assume that the teams are totally ordered w.r.t. importance.

Optimal Sequence with Homogeneous Team Size

Assumption 2.

For any subsets of agents S, S' and T such that S is more important than S' and $T \cap (S \cup S') = \emptyset$, we have

$$p(S \cup T) - p(S) \geq p(S' \cup T) - p(S').$$

Proposition 7.

The optimal sequence π^* satisfies that

(i) *In each star, successors are more important than predecessors;*

(ii) *Each successor in star i is more important than every predecessor in each star that moves after i.*

(iii) *If Assumption 2 holds, then the stars move in ascending order of importance; thus, m_i^* is nondecreasing along π^*.*

Simple Algorithm for Highly Differential Importance

Corollary 5.

Given Assumption 2. Suppose for any two stars i and j such that i is more important than j, $r_i^{\hat{n}-1}$ is more important than r_j^1, then π^* can be fully characterized by the following algorithm:

1. *Align the stars in ascending order of importance, and set each star i such that $m_i = 0$;*
2. *In the last star, make the most important peripheral agent a successor, then the second important peripheral agent, etc. When every peripheral agent becomes successor, move backward one star and continue this procedure;*
3. *Continue until the total rewards start to increase.*

Summary

We studied a team incentive design problem in which

- Agents are located on a network and work on a joint project.
- Principal chooses the work sequence and the reward scheme.
- Network and sequence jointly determine internal information.
- Principal seeks the least-cost effort-inducing mechanism.

Main results

- Under complementarity, action transparency motivates efforts.
- Effectiveness of the transparency on incentives is diminishing.
- Agents work sequentially and more important ones work later.
- Larger sub-teams are assigned to either end of the sequence.
- Monitors account for a small share and cluster in later stages.