Competitive Nonlinear Pricing for Signals

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Introduction

Lu (2018) studies monopolistic nonlinear pricing for signals

- When receivers observe price schedule: downward distortion.
- When receivers do not observe price schedule: higher quantity.

This paper studies competitive nonlinear pricing for signals

- Solves the optimal pricing under different market structures.
- Assesses the effects of pricing transparency on signaling.

Main results

- In the monopoly observed case, there is either downward distortion, or full efficiency, or upward distortion at low end.
- In the monopoly unobserved case, quantity is more dispersed.
- Competition results in higher quantity and market coverage.

Literature

Classic signaling and screening models

- Spence (1973), Mussa and Rosen (1978), Maskin and Riley (1984).

Monopolistic nonlinear pricing for signals

- Rayo (2013), Lu (2018).

Competitive nonlinear pricing for non-signals

- Gilbert and Matutes (1993), Verboven (1999), Villas-Boas and Schmidt-Mohr (1999), Armstrong and Vickers (2001), Rochet and Stole (2002), Ellison (2005), Yang and Ye (2008).

Multi-dimensional screening models

- McAfee and McMillan (1988), Armstrong (1996), Rochet and Chonè (1998), Armstrong and Rochet (1999), Thanassoulis (2004).

Model

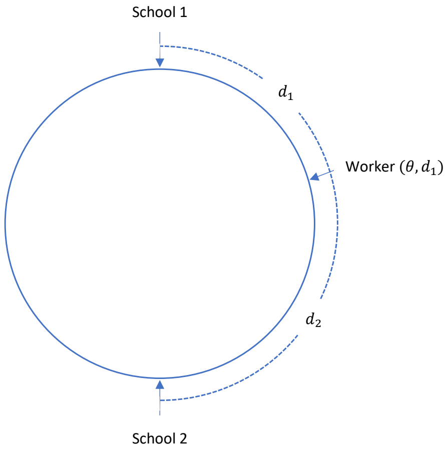
Model

Players and actions

- Two schools post tuition schemes $T_i(z)$, $i = 1, 2$.
- A worker has a two-dimensional preference over education:
  Vertical type $\theta \sim U[0, 1]$; horizontal type $d_i \sim U[0, \frac{1}{2}]$.
- Worker chooses at most one school to attend;
  Obtains productivity $Q(z, \theta) = \gamma\theta z + z$, where $\gamma > 0$.
- A competitive labor market posts wages $W_i(z)$, $i = 1, 2$.

Information

- Worker's type $(\theta, \{d_i\})$ is privately known.
- Worker's education choice (i, z) is publicly observed.
- In the *observed* case, employers observe both $T_i(z)$.
- In the *unobserved* case, employers observe neither of $T_i(z)$.



Locations: Salop's Circle Model

Payoffs

- School i's expected profit $\Pi_i = \mathbb{E}[T_i(z(\theta))]$.
- Worker's gross utility by attending school i

$$V_i(z, \theta) = W_i(z) - T_i(z) - C(z, \theta),$$

  where $C(z, \theta) = z^2 + (1 - \theta)z$ is the effort cost.
- Worker obtains zero utility if purchases no education.
- Surplus: $S(z, \theta) = (\gamma + 1)\theta z - z^2$; first-best: $z^{fb}(\theta) = \frac{(\gamma+1)\theta}{2}$.

Worker's problem

- Worker incurs a *transport cost* $kd_i \geq 0$ if attends school i.
- Worker chooses $i \in \{\emptyset, 1, 2\}$ and z to maximize net utility

$$U_i(z, \theta, d_i) = V_i(z, \theta) - kd_i.$$

A Direct Mechanism

Timing

- School i offers a contract $\langle z_i(\theta), T_i(z) \rangle$ to worker.
- Labor market posts $W_i(z)$ based on the observability of offers.
  Market belief is correct if $W_i(z) = \mathbb{E}[Q(z, \theta) | z_i(\theta)]$.
- Worker reports a type $\tilde{\theta}$ to only the school he attends.

School i's problem

- In the observed case, max. Π_i s.t. *IC*, *IR* and correct belief.
- In the unobserved case, max. Π_i s.t. *IC* and *IR*.
- Focus on the *symmetric school-optimal separating equilibrium*.

Preliminaries

Lemma 1.

In both cases, an allocation $\langle z(\theta), V(\theta)\rangle$ is IC if

(i) $z(\theta)$ *is non-decreasing.*

(ii) *Define* $\theta_0 := \inf\{\theta | z(\theta) > 0\}$, *then for all* $\theta > \theta_0$,

$$V(\theta) = V(\theta_0) + \int_{\theta_0}^{\theta} -C_\theta(z(s), s)ds = V(\theta_0) + \int_{\theta_0}^{\theta} z(s)ds.$$

Market share

- School i's market share for type θ is twice

$$s_i(\theta) := \min\left\{\frac{V_i(\theta)}{k}, \frac{1}{4} + \frac{V_i(\theta) - V_{-i}(\theta)}{2k}\right\}.$$

- The switching type θ_1 is given by

$$V_1(\theta_1) + V_2(\theta_1) = \frac{k}{2}.$$

A Bertrand-Spence Benchmark

- Suppose $k = 0$, Bertrand competition leads to $T_i(z) \equiv 0$.
- The model is translated to a Spence's signaling game.
- In the least-cost separating equilibrium, education:

$$z^s(\theta) = \frac{(2\gamma+1)\theta}{2} \geq \frac{(\gamma+1)\theta}{2} = z^{fb}(\theta).$$

- Define the *signaling intensity* as

$$\frac{z^s(\theta) - z^{fb}(\theta)}{z^{fb}(\theta)} = \frac{\gamma}{\gamma+1}.$$

- The greater γ is, the more intense signaling activity is.

Monopoly

The Observed Case

- A monopolist maximizes the joint profit of both schools.

Monopolist's problem

- The monopolist solves

$$\max_{z(\theta)} \underbrace{\int_{\theta_0}^{\theta_1} [S(z,\theta) - V(\theta)] \frac{V(\theta)}{k} d\theta}_{\text{Phase I: partially covered range}} + \underbrace{\int_{\theta_1}^{1} [S(z,\theta) - V(\theta)] \frac{1}{4} d\theta}_{\text{Phase II: fully covered range}}$$

$$s.t. \; V'(\theta) = z(\theta), \;\; z'(\theta) \geq 0, \;\; V(\theta_1) = \frac{k}{4}.$$

- If $\theta_0 \in (0,1]$, then $V(\theta_0) = 0$; otherwise, $V(\theta_0)$ is free.

Solving Monopolist's Problem

- Define the *Hamiltonian* for each phase as

$$\text{Phase I: } H_1 = [S(z,\theta) - V(\theta)]\frac{V(\theta)}{k} + \lambda z.$$

$$\text{Phase II: } H_2 = [S(z,\theta) - V(\theta)]\frac{1}{4} + \lambda z.$$

- Applying the *Maximum Principle*, for each phase $j = 1, 2$,

$$\frac{\partial H_j}{\partial z} = 0 \text{ and } \dot{\lambda}(\theta) = -\frac{\partial H_j}{\partial V} \text{ with } \lambda(1) = 0.$$

- Solving Phase I yields an ODE

$$(\gamma + 3)V - 2\ddot{V}V - \dot{V}^2 = 0. \tag{1}$$

- Solving Phase II, we have $z(\theta) = \frac{(\gamma+2)\theta - 1}{2}$ for $\theta \in [\theta_1, 1]$.

Determing cutoff types

- Suppose $\theta_0 > 0$, then from (1), for $\theta \in [\theta_0, \theta_1)$,

$$V(\theta) = \frac{\gamma+3}{8}(\theta-\theta_0)^2, \ z(\theta) = \frac{\gamma+3}{4}(\theta-\theta_0).$$

- Applying smooth pasting,

$$\theta_0 = \frac{1}{\gamma+2} - \frac{(\gamma+1)\sqrt{2(\gamma+3)k}}{2(\gamma+2)(\gamma+3)}, \tag{2}$$

$$\theta_1 = \frac{1}{\gamma+2} + \frac{\sqrt{2(\gamma+3)k}}{2(\gamma+2)}. \tag{3}$$

- Thus, $\theta_0 > 0$ if $k < \frac{2(\gamma+3)}{(\gamma+1)^2}$; $\theta_1 < 1$ if $k < \frac{2(\gamma+1)^2}{(\gamma+3)}$.

Optimal Contract — Low Signaling Intensity

Proposition 1.

When $\gamma \leq 1$, equilibrium exists. The optimal contract satisfies:

- *If $k \leq \frac{2(\gamma+1)^2}{(\gamma+3)}$,*

$$z^{m_o}(\theta) = \begin{cases} \frac{\gamma+3}{4}(\theta - \theta_0^{m_o}) & \text{if } \theta_0^{m_o} \leq \theta < \theta_1^{m_o} \\ \frac{(\gamma+2)\theta}{2} - \frac{1}{2} & \text{if } \theta_1^{m_o} \leq \theta \leq 1, \end{cases}$$

  where $\theta_0^{m_o}$ and $\theta_1^{m_o}$ are given by (2) and (3), respectively.

- *If $k > \frac{2(\gamma+1)^2}{(\gamma+3)}$, $\theta_0^{m_o} = \frac{1-\gamma}{\gamma+3}$,*

$$z^{m_o}(\theta) = \frac{\gamma+3}{4}\left(\theta - \theta_0^{m_o}\right) \text{ if } \theta_0^{m_o} \leq \theta \leq 1.$$

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  where $\theta_0^{m_o}$ *and* $\theta_1^{m_o}$ *are given by (2) and (3), respectively.*

- *If* $k > \frac{2(\gamma+1)^2}{(\gamma+3)}$, $\theta_0^{m_o} = \frac{1-\gamma}{\gamma+3}$,

$$z^{m_o}(\theta) = \frac{\gamma+3}{4}\left(\theta - \theta_0^{m_o}\right) \text{ if } \theta_0^{m_o} \leq \theta \leq 1.$$

- *If* $\gamma < 1$, $\theta_0^{m_o} > 0$ *and* $z^{m_o}(\theta) < z^{fb}(\theta)$ *on* $[0, 1)$.

Optimal Contract — Low Signaling Intensity

Proposition 1.

When $\gamma \leq 1$, equilibrium exists. The optimal contract satisfies:

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  where $\theta_0^{m_o}$ *and* $\theta_1^{m_o}$ *are given by (2) and (3), respectively.*

- *If* $k > \frac{2(\gamma+1)^2}{(\gamma+3)}$, $\theta_0^{m_o} = \frac{1-\gamma}{\gamma+3}$,

$$z^{m_o}(\theta) = \frac{\gamma+3}{4}\left(\theta - \theta_0^{m_o}\right) \text{ if } \theta_0^{m_o} \leq \theta \leq 1.$$

- *If* $\gamma < 1$, $\theta_0^{m_o} > 0$ *and* $z^{m_o}(\theta) < z^{fb}(\theta)$ *on* $[0, 1)$.
- *If* $\gamma = 1$ *and* $k \geq \frac{2(\gamma+1)^2}{(\gamma+3)}$, $z^{m_o}(\theta) = z^{fb}(\theta)$ *on* $[0, 1]$.

Optimal Contract — High Signaling Intensity

Vertical market is partially covered

- When $\gamma > 1$, if $k \leq \frac{2(\gamma+3)}{(\gamma+1)^2}$, $\theta_0^{m_o} \geq 0$ and $\theta_1^{m_o} < 1$.
- The optimal contract is also given by Proposition 1.

Vertical market is fully covered

- When $\gamma > 1$, if $k > \frac{2(\gamma+3)}{(\gamma+1)^2}$, $\theta_0^{m_o} = 0$, meaning $\lambda(0) = 0$.
- Solving Phase I is equivalent to solving the program

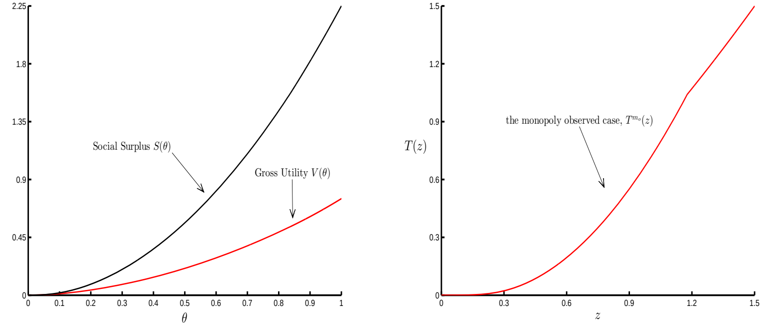
$$(\gamma + 3)V - 2\ddot{V}V - \dot{V}^2 = 0$$

$$s.t.\ \dot{V}(0) = 0,\ V(\theta_1) = \frac{k}{4},\ \dot{V}(\theta_1) = \frac{(\gamma+2)\theta_1 - 1}{2}.$$

- Solving Phase II follows exactly the previous steps.

A Numerical Solution

(a) Gross Utility (b) Tuition Scheme



Assumption: $\gamma = 2$ and $k = 2$

Over-Education at Low End

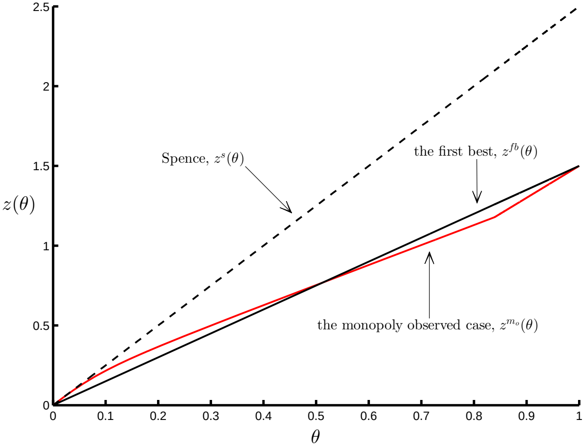
Proposition 2.

When $\gamma > 1$, for sufficiently large k, there is a cutoff $\tilde{\theta} \in (0, \theta_1^{m_o})$, such that $z^{m_o}(\theta) > z^{fb}(\theta)$ on $(0, \tilde{\theta})$; $z^{m_o}(\theta) < z^{fb}(\theta)$ on $(\tilde{\theta}, 1)$. Furthermore, $z^{m_o}(\theta) < z^{s}(\theta)$ on $(0, 1]$.

Idea

- Monopolist cuts prices at low end to gain market share.
- Signaling and market penetration outweigh screening.

A Numerical Solution (cont.)



Assumption: $\gamma = 2$ and $k = 2$

The Effects of Horizontal Differentiation

Corollary 1.

When $\theta_0^{m_o} > 0$ *and* $\theta_1^{m_o} < 1$, *as* k *increases:*

(i) $V^{m_o}(\theta)$ *increases on* $(\theta_0^{m_o}, 1]$.

(ii) $z^{m_o}(\theta)$ *increases on* $(\theta_0^{m_o}, \theta_1^{m_o}]$; *remains the same on* $(\theta_1^{m_o}, 1]$.

(iii) *The market coverage* $[\theta_0^{m_o}, 1]$ *extends, but* $[\theta_1^{m_o}, 1]$ *shrinks.*

Idea

- Monopolist provides more *rents* to gain market share.
- This can be achieved by more education or larger coverage.
- The optimal allocation requires a balance between these two.

The Unobserved Case

Monopolist's problem

- Given $W(z)$, the monopolist solves

$$\max_{z(\theta)} \underbrace{\int_{\theta_0}^{\theta_1} [W(z) - C(z,\theta) - V(\theta)] \frac{V(\theta)}{k} d\theta}_{\text{Phase I: partially covered range}}$$

$$+ \underbrace{\int_{\theta_1}^{1} [W(z) - C(z,\theta) - V(\theta)] \frac{1}{4} d\theta}_{\text{Phase II: fully covered range}}$$

$$s.t.\ V'(\theta) = z(\theta),\ z'(\theta) \geq 0,\ V(\theta_1) = \frac{k}{4}.$$

- If $\theta_0 \in (0,1]$, then $V(\theta_0) = 0$; otherwise, $V(\theta_0)$ is free.
- In equilibrium, $W(z) = \mathbb{E}[Q(z,\theta)|z(\theta)]$ using Bayes' rule.

Solving Monopolist's Problem

- Define the Hamiltonian for each phase as

$$\text{Phase I: } H_1 \equiv [W(z) - C(z,\theta) - V(\theta)]\frac{V(\theta)}{k} + \lambda z.$$

$$\text{Phase II: } H_2 \equiv [W(z) - C(z,\theta) - V(\theta)]\frac{1}{4} + \lambda z.$$

- In equilibrium, $W'(z) = Q_z(z,\theta) + Q_\theta(z,\theta) \cdot \theta'(z)$.
- Solving Phase I yields an ODE

$$(2\gamma + 3)V - \frac{\gamma V \dot{V} \dddot{V}}{\ddot{V}^2} - 2\ddot{V}V + \frac{\gamma \dot{V}^2}{\ddot{V}} - \dot{V}^2 = 0. \qquad (4)$$

- Solving Phase II with the desired initial condition, we have

$$W(z) = \frac{\gamma}{\gamma+1}z^2 + \frac{2(\gamma+1)}{\gamma+2}z.$$

- This implies $z(\theta) = (\gamma+1)(\theta - \frac{1}{\gamma+2})$ for $\theta \in [\theta_1, 1]$.

Determing cutoff types

- Suppose $\theta_0 > 0$, then from (4), for $\theta \in [\theta_0, \theta_1)$,

$$V(\theta) = \frac{4\gamma + 3}{8}(\theta - \theta_0)^2, \; z(\theta) = \frac{4\gamma + 3}{4}(\theta - \theta_0).$$

- Applying smooth pasting,

$$\theta_0 = \frac{1}{\gamma + 2} - \frac{\sqrt{2(4\gamma + 3)k}}{4(\gamma + 1)(4\gamma + 3)} \tag{5}$$

$$\theta_1 = \frac{1}{\gamma + 2} + \frac{\sqrt{2(4\gamma + 3)k}}{4(\gamma + 1)} \tag{6}$$

- It turns out that $\theta_0 > 0$ and $\theta_1 \leq 1$ always holds.

Optimal Contract

Proposition 3.

For all $\gamma > 0$, equilibrium exists. The optimal contract satisfies:

- *If $k \leq \frac{8(\gamma+1)^4}{(4\gamma+3)(\gamma+2)^2}$,*

$$z^{m_u}(\theta) = \begin{cases} \frac{4\gamma+3}{4}(\theta - \theta_0^{m_u}) & \text{if } \theta_0^{m_u} \leq \theta < \theta_1^{m_u} \\ (\gamma+1)\theta - \frac{\gamma+1}{\gamma+2} & \text{if } \theta_1^{m_u} \leq \theta \leq 1, \end{cases}$$

  where $\theta_0^{m_u}$ and $\theta_1^{m_u}$ are given by (5) and (6), respectively.

- *If $k > \frac{8(\gamma+1)^4}{(4\gamma+3)(\gamma+2)^2}$, $\theta_0^{m_u} = \frac{1}{2\gamma+3}$,*

$$z^{m_u}(\theta) = \frac{4\gamma+3}{4}(\theta - \theta_0^{m_u}) \text{ if } \theta_0^{m_u} \leq \theta \leq 1.$$

Educatoin Levels are More Dispersed

Proposition 4.

(i) *For all* $\gamma, k > 0$, $\theta_0^{m_u} > \theta_0^{m_o}$ *and* $\theta_1^{m_u} < \theta_1^{m_o}$.

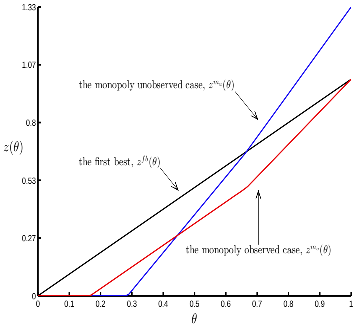
(ii) *There is a cutoff* $\tilde{\theta} \in (\theta_0^{m_u}, \theta_1^{m_u})$, *such that* $z^{m_u}(\theta) < z^{m_o}(\theta)$ *on* $(\theta_0^{m_o}, \tilde{\theta})$; $z^{m_u}(\theta) > z^{m_o}(\theta)$ *on* $(\tilde{\theta}, 1]$. *The length of the interval* $(\theta_0^{m_o}, \tilde{\theta})$ *is increasing in* k, *and vanishes as* $k \to 0$.

Idea

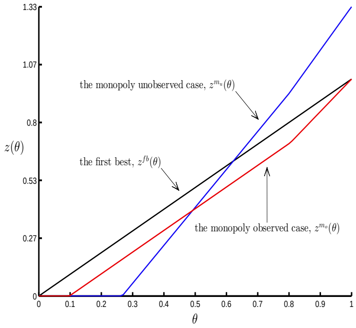
- Education demand is more elastic due to signal jamming.
- Monopolist supplies more education in fully covered range.
- But it is not as profitable in partially covered range due to IC.

Comparison of Education Levels

(a) $\gamma = 1,\ k = 0.5$



(b) $\gamma = 1,\ k = 1$



Consumer Surplus and Tuition

Proposition 5.

(i) *There is a cutoff* $\tilde{\theta} \in (\theta_0^{m_u}, 1)$*, such that* $V^{m_u}(\theta) < V^{m_o}(\theta)$ *on* $(\theta_0^{m_o}, \tilde{\theta})$*;* $V^{m_u}(\theta) > V^{m_o}(\theta)$ *on* $(\tilde{\theta}, 1]$*. The length of the interval* $(\theta_0^{m_o}, \tilde{\theta})$ *is increasing in* k*, and vanishes as* $k \to 0$*.*

(ii) *There is a cutoff* $\tilde{z} \in (0, z^{fb}(1))$*, such that* $T^{m_u}(z) > T^{m_o}(z)$ *on* $(0, \tilde{z})$*;* $T^{m_u}(z) > T^{m_o}(z)$ *on* $(\tilde{z}, z^{fb}(1)]$*. The length of the interval* $(0, \tilde{z})$ *is increasing in* k*, and vanishes as* $k \to 0$*.*

Implication

- A lower-type who is close to either school benefits more from the rise in horizontal differentiation in the observed case.

Duopoly

The Observed Case

- Focus on the case in which $\theta_0 > 0$ under monopoly.

School's problem

- Given the other school's contract, school i solves

$$\underbrace{\max_{z_i(\theta)} \int_{\theta_0}^{\theta_1} [S(z,\theta) - V_i(\theta)] \frac{V_i(\theta)}{k} d\theta}_{\text{Phase I: local monopoly range}}$$

$$+ \underbrace{\int_{\theta_1}^{1} [S(z,\theta) - V_i(\theta)] \left[\frac{1}{4} + \frac{V_i(\theta) - V_{-i}(\theta)}{2k}\right] d\theta}_{\text{Phase II: competition range}}$$

$$s.t.\ V'(\theta) = z(\theta),\ z'(\theta) \geq 0,\ V(\theta_1) = \frac{k}{4}.$$

- If $\theta_0 \in (0, 1]$, then $V_i(\theta_0) = 0$; otherwise, $V_i(\theta_0)$ is free.

Solving School's Problem

- Define the *Hamiltonian* for each phase as

$$\text{Phase I: } H_1 = [S(z,\theta) - V_i(\theta)]\frac{V_i(\theta)}{k} + \lambda z.$$

$$\text{Phase II: } H_2 = [S(z,\theta) - V_i(\theta)]\left[\frac{1}{4} + \frac{V_i(\theta) - V_{-i}(\theta)}{2k}\right] + \lambda z.$$

- Solving Phase I follows exactly the same steps of monopoly.
- Solving Phase II is equivalent to solving the program

$$\ddot{V} = \frac{\gamma+2}{2} + \frac{1}{k}[V - (\gamma+1)\theta\dot{V} + \dot{V}^2] \tag{7}$$

$$s.t.\ V(\theta_1) = \frac{k}{4}, \dot{V}(\theta_1) = \frac{\gamma+3}{4}(\theta_1 - \theta_0), \dot{V}(1) = \frac{\gamma+1}{2}.$$

Optimal Contract

Proposition 6.

Equilibrium exists. The optimal contract satisfies:

$$z^{d_o}(\theta) = \begin{cases} \frac{\gamma+3}{4}(\theta - \theta_0^{d_o}) & \text{if } \theta_0^{d_o} \leq \theta < \theta_1^{d_o} \\ \dot{V}^{d_o}(\theta) & \text{if } \theta_1^{d_o} \leq \theta \leq 1, \end{cases}$$

where $V^{d_o}(\theta)$ *and* $\theta_1^{d_o}$ *are given by (7), and* $\theta_0^{d_o} = \theta_1^{d_o} - \sqrt{\frac{2k}{\gamma+3}}$.

Equilibrium Discontinuity

- Equilibrium is discontinuous at $k = 0$.
- It is always optimal to exclude sufficiently low types if $k > 0$.

Education Levels and Market Coverage

Proposition 7.

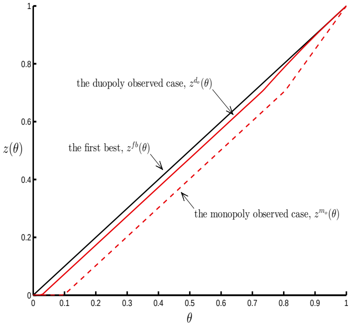
Suppose $\theta_0^{m_o} > 0$, $\theta_0^{d_o} < \theta_0^{m_o}$ *and* $z^{d_o}(\theta) > z^{m_o}(\theta)$ *for* $\theta \in (\theta_0^{d_o}, 1)$. *In contrast to the monopoly case, more worker types (in terms of both vertical and horizontal type) receive education, and each participating type obtains higher net utility.*

Idea

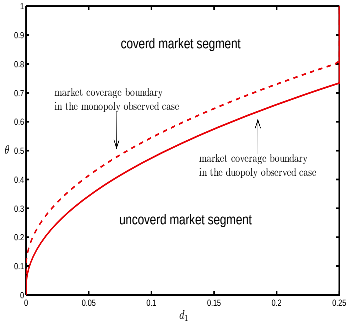
- Competition leads to more education in competition range.
- This relaxes IC, thus lower types also receive more education.

The Effects of Market Competition

(a) Education Supply



(b) Market Coverage



Assumption: $\gamma = 1$ and $k = 1$

The Unobserved Case

School's problem

- Given $W(z)$ the other school's contract, school i solves

$$\max_{z_i(\theta)} \underbrace{\int_{\theta_0}^{\theta_1} [W(z) - C(z,\theta) - V_i(\theta)] \frac{V_i(\theta)}{k} d\theta}_{\text{Phase I: local monopoly range}}$$

$$+ \underbrace{\int_{\theta_1}^{1} [W(z) - C(z,\theta) - V_i(\theta)] \left[\frac{1}{4} + \frac{V_i(\theta) - V_{-i}(\theta)}{2k}\right] d\theta}_{\text{Phase I: competition range}}$$

$$s.t.\ V'(\theta) = z(\theta),\ z'(\theta) \geq 0,\ V(\theta_1) = \frac{k}{4}.$$

- If $\theta_0 \in (0,1]$, then $V_i(\theta_0) = 0$; otherwise, $V_i(\theta_0)$ is free.
- In equilibrium, $W(z) = \mathbb{E}[Q(z,\theta)|z(\theta)]$ using Bayes' rule.

Solving School's Problem

- Define the Hamiltonian for each phase as

$$\text{Phase I: } H_1 = [W(z) - C(z,\theta) - V_i(\theta)]\frac{V_i(\theta)}{k} + \lambda z.$$

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- Solving Phase I follows exactly the same steps of monopoly.
- Solving Phase II is equivalent to solving the program

$$\dddot{V} = \frac{(\gamma+2)\ddot{V} + (\gamma-2)\ddot{V}^2}{\gamma \dot{V}} + \frac{2}{\gamma k}[\frac{V\ddot{V}}{\dot{V}} - (\gamma+1)\theta\dot{V} + \dot{V}\ddot{V}] \quad (8)$$

$$s.t.\ V(\theta_1) = \frac{k}{4},\ \dot{V}(\theta_1) = \frac{\sqrt{2(4\gamma+3)k}}{4},\ \frac{[\gamma - 2\ddot{V}(1)]\dot{V}(1)}{\ddot{V}(1)} = \gamma+1.$$

Optimal Contract

Proposition 8.

Equilibrium exists. The optimal contract satisfies:

$$z^{d_u}(\theta) = \begin{cases} \frac{4\gamma+3}{4}(\theta - \theta_0^{d_u}) & \text{if } \theta_0^{d_u} \leq \theta < \theta_1^{d_u} \\ \dot{V}^{d_u}(\theta) & \text{if } \theta_1^{d_u} \leq \theta \leq 1, \end{cases}$$

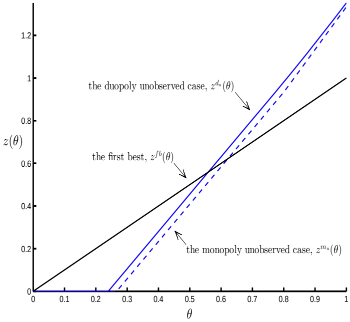
where $V^{d_u}(\theta)$ *and* $\theta_1^{d_u}$ *are given by (8), and* $\theta_0^{d_u} = \theta_1^{d_u} - \sqrt{\frac{2k}{4\gamma+3}}$.

Equilibrium Discontinuity

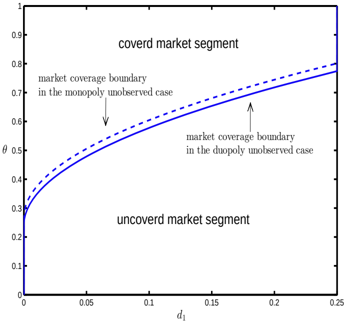
- Equilibrium is discontinuous at $k = 0$.
- It is always optimal to exclude sufficiently low types if $k > 0$.

The Effects of Market Competition

(a) Education Supply



(b) Market Coverage



Assumption: $\gamma = 1$ and $k = 1$

Summary

In this paper

- We studied competitive nonlinear pricing for signals.
- We solved the optimal pricing for different market structures.

Main results

- In the monopoly observed case, whether there is downward distortion, or full efficiency, or upward distortion depends on the degrees of signaling intensity and horizontal differentiation.
- In the monopoly unobserved case, signal quantities are more dispersed due to signal jamming and market penetration.
- Market competition results in a higher market coverage and larger quantities for both the observed and unobserved case.